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1 SEM TDC GEMT (CBCS) GE 1 (A/B/C)

2023

(November)

MATHEMATICS

(Generic Elective)

Paper : GE–1

The figures in the margin indicate full marks for the questions

Paper : GE–1 (A)

(Differential Calculus)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

1. Answer the following (any *one*) : 1

(a) Define limit of a function.

(b) Write down the value of $D^n(\log ax)$.

2. Find the value of the following (any *two*) : 2×2=4

(i) $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

(ii) $\lim_{x \to \infty} (x - \sqrt{x^2 + x})$

(iii) $\lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$

3. *(a)* A function $f(x)$ is defined as follows :

$$f(x) = x \sin \frac{1}{x}; \quad x \neq 0$$
$$= 0; \quad x = 0$$

Show that $f(x)$ is continuous at $x = 0$. 3

Or

A function $f(x)$ is defined as follows :

$$f(x) = \frac{1}{2} - x \quad \text{when} \quad 0 < x < \frac{1}{2}$$
$$= \frac{1}{2} \quad \text{when} \quad x = \frac{1}{2}$$
$$= \frac{3}{2} - x \quad \text{when} \quad \frac{1}{2} < x < 1$$

Show that $f(x)$ is discontinuous at $x = \frac{1}{2}$.

(b) Find *n*th derivative of the following (any *one*) : 3

(i) $y = \frac{1}{\sqrt{x}}$

(ii) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

4. State Leibnitz's theorem. Using Leibnitz's theorem, prove that if $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$; $|x| < 1$, then

$$(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$$

1+3=4

Or

If $y = \tan^{-1} x$, then prove that

$$(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$$ 4

5. *(a)* Define homogeneous function. State and prove Euler's theorem on homogeneous functions. 1+4=5

(b) If

$$u = f(x, y) = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$$

then prove that

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$$

5

Or

If

$$u = \log(x^3 + y^3 + z^3 - 3xyz)$$

then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^2}$$

6. Answer the following (any *one*) : 1

(a) Write down the equation of tangent parallel to *x*-axis.

(b) Define curvature.

7. Find the radius of curvature of the parabola $y^2 = 4x$ at the vertex (0, 0). 2

Or

Find the radius of curvature at the point θ on the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

8. *(a)* Find the asymptotes of the cubic expression

$$x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$$ 4

(b) Answer any *one* of the following : 4

(i) Define singular points. Examine the curve $y^2(1 + x) = x^2(1 - x)$ for singular points at the origin.

(ii) Discuss about singular points of an algebraic curve at the origin.

9. Determine the position and nature of multiple points of the curve

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$$ 4

10. Answer any *two* of the following : 5×2=10

(a) Trace the curve $ay^2 = x^3$.

(b) Sketch the curve $r = a(1 - \cos\theta)$.

(c) Trace the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$.

11. *(a)* Is Rolle's theorem, applicable to the function $f(x) = \tan x$ in the interval $[0, \pi]$? Give reasons. 1

(b) State L' Hôspital's rule. 1

12. *(a)* Find the value of c in the mean value theorem $f(b) - f(a) = (b - a) f'(c)$, if $f(x) = x^2$, $a = 1$, $b = 2$. 2

(b) Answer the following (any *one*) : 2

(i) State Taylor's theorem with Lagrange's form of remainder after n terms.

(ii) Find

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$$

13. *(a)* State and prove Cauchy's mean value theorem. 3

(b) Find the maximum value of $(x - 1)(x - 2)(x - 3)$. 3

14. *(a)* Stating Maclaurin's series, prove that

$$\cos x = 1 - \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} - \cdots \infty$$ 4

Or

Show that the function

$$f(x, y) = x^2 - 3xy^2 + 2y^4$$

has neither a maximum nor a minimum value at the origin.

(b) Evaluate the following (any *one*) : 4

(i) $\lim_{x \to 0} \frac{x^2 + 2\cos x - 2}{x \sin^3 x}$

(ii) $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^2 \sin x}$

15. *(a)* Expand the following function in powers of x in infinite series : 5

(i) $f(x) = \log(1 + x)$

Or

(ii) $f(x) = e^{mx}$

(b) State and prove the Taylor's theorem. 5

Or

Illustrate the geometrical interpretation of Lagrange's mean value theorem.

Paper : GE–1 (B)

(Object-Oriented Programming in C++)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer the following (any *five*) : 2×5=10

(a) State any two forms of inheritance.

(b) State two differences between C and C++.

(c) Write two characteristics of object-oriented programming language.

(d) Explain the use of friend function with the help of example.

(e) Write down the rules for an identifier.

(f) Write down the syntax and example to create a class.

2. Answer the following (any *five*) : 3×5=15

(a) Explain the use of the following operators :

cout, new, delete

(b) Write down the rules of operator overloading for binary operators.

(c) How do we declare a member of a class static?

(d) State the difference between static and dynamic binding.

(e) Explain the structure of a C++ program.

(f) "Overloaded constructor is the copy constructor." Comment on it.

3. Answer the following (any *five*) : 4×5=20

(a) Explain briefly the postfix and prefix operators.

(b) State the difference between function overloading and function overriding.

(c) Define a class representing following members as 'Bus' :

Data members	*Member function*
—Bus number	—Initialise member
—Bus name	—Input Bus data
—Source	—Display data
—Destination	
—Journey date	

(Continued)

(d) Differentiate between compile-time polymorphism and run-time polymorphism.

(e) Write a C++ program to copy contents of ABC.txt to XYZ.txt.

(f) Write a C++ program to display the number of objects created using static member.

4. Answer the following (any *three*) : 5×3=15

(a) Write a C++ program to create data file containing list of marks obtained by students

John 352

Hari 562

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Use a class object to store each set of data.

(b) Write a C++ program to add two complex numbers using '+' operator.

(c) Define 'operator overloading'. List the operators that cannot be overloaded.

(d) Can base class access members of a derived class? Give reasons.

(Turn Over)

Paper : GE–1 (C)

(Finite Element Method)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. *(a)* State True or False : 1

   Each finite element is viewed as an independent domain itself.

   (b) Write one advantage of subdivision of whole domain into parts in finite element method. 1

   (c) Write about finite element discretization. 2

   (d) Write on which accuracy and convergence of finite element solution depends. 2

   (e) Write one difference between Ritz method and Galerkin method. 2

   (f) Describe residual function. 4

   (g) Describe Ritz variational method. 8

   Or

   Construct the weak forms and quadratic functionals of the equation

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right)+f=0, \qquad 0<x<L$$

$$\left(u\frac{du}{dx}\right)_{x=0}=0, \qquad u(1)=\sqrt{2},$$

2. *(a)* Define order of an element. 2

   (b) By taking a mesh of four linear elements, write the assembled set of equations for

$$-\frac{d^2u}{dx^2}-u+x^2=0, \qquad \text{for } 0<x<1$$

$$u(0)=0, \quad u(1)=0$$

10

   Or

   Solve

$$-\frac{d^2u}{dx^2}=\cos\pi x, 0<x<1, u(0)=0, u(1)=0$$

   using uniform mesh of three linear elements.

3. *(a)* Define shape function. 2

   (b) Solve

$$\frac{d^2y}{dx^2}+x=0, 0<x<1, y(0)=y(1)=0$$

   using Ritz method (taking two base functions). 10

   Or

   Solve

$$\frac{d^2y}{dx^2}-x=0, y(0)=0, y'(1)=\frac{1}{2}$$

   by Ritz method.

4. *(a)* Write how accuracy in finite element method may be increased. 2

(b) Describe the finite element model of Poisson equation. 10

Or

Describe triangular element mesh assembly.

5. *(a)* Describe about approximating functions. 4

Or

Explain the use of triangular element.

(b) Write the steps involved in the finite element analysis of an ideal problem. 8

6. Solve : 12

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad u(x, 0) = \cos\frac{\pi x}{2},$$

$$-1 \le x \le 1$$

$$u = 0, \quad x = \pm 1, \quad t > 0$$

Or

Solve :

$$\frac{\partial u}{\partial t} - \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = f, \ \frac{\partial u}{\partial x}(0, y, t) = 0, \ \frac{\partial u}{\partial y}(x, 0, t) = 0$$

$$u(x, 1, t) = 0, \quad u(1, y, t) = 0, \quad u(x, y, 0) = 0$$

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