

**6 SEM TDC MTMH (CBCS) C 13**

**2024**

( May )

**MATHEMATICS**

( Core )

Paper : C-13

**( Metric Spaces and Complex Analysis )**

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) Write the triangle inequality of metric space. 1
- (b) A metric  $d$  on a non-empty set may be negative. State True or False. 1
- (c) A metric space consists of two objects. Write that objects. 2
- (d) Define a pseudometric on a non-empty set. 2
- (e) Define a complete metric space. 2

(f) Answer any two from the following :

6×2=12

(i) Show that in any metric space  $X$ , each open sphere is an open set.

(ii) Let  $X$  be a metric space with metric  $d$ . Show that  $d_1$  defined by

$$d_1 = \frac{d(x, y)}{1 + d(x, y)}$$

is also a metric on  $X$ .

(iii) Show that a Cauchy sequence is convergent if and only if it has a convergent subsequence.

(iv) Show that a subset of a metric space is bounded if and only if it is non-empty and is contained in some closed sphere.

2. (a) Write when a metric space is called sequentially compact. 1
- (b) Write an example of an uniformly continuous function in a metric space. 1
- (c) Define a continuous mapping in a metric space. 2
- (d) Show that the homeomorphism on the set of all metric spaces is an equivalence relation. 5

Or

Let  $f$  is a continuous mapping of a metric space  $X$  into a metric space  $Y$ . Then show that if  $E$  is a connected subset of  $X$ , then  $f(E)$  is connected.

(e) Let  $X$  and  $Y$  be metric spaces and  $f$  a mapping of  $X$  into  $Y$ . Then show that  $f$  is continuous at  $x_0$  if and only if  $x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow f(x_0)$ . 6

Or

Show that every compact metric space has the Bolzano-Weierstrass property.

3. (a) Define extended complex plane. 1
- (b) If a function  $f$  is continuous throughout a region  $R$ , then it is not bounded. State True or False. 1
- (c) Show that  $\lim_{z \rightarrow 0} \frac{z}{\bar{z}}$  does not exist. 2
- (d) Find the arg  $z$ , where  $z = \frac{-5}{1+i\sqrt{2}}$ . 2
- (e) Show that  $\frac{dw}{dz} = (\cos\theta - i\sin\theta) \frac{\partial w}{\partial r}$ ,  $w = w(r, \theta)$  is an analytic function. 4

Or

Let  $f(z) = z - \bar{z}$ . Show that  $f'(z)$  does not exist at any point.

- (f) Describe the mapping  $w = z^2$ . 5
4. (a) Find the analytic function  $f(z) = u + iv$ , where  $u(x, y) = \sinh x \sin y$ . 5
- (b)  $e^z$  may have negative value. State True or False. 1

- (c) Show that  $\log(e^z) = z + 2n\pi i$ ,  $n = 0, 1, 2, \dots$  4

Or

Evaluate  $\int_C \bar{z} dz$ , where  $C$  is the right-hand half of the circle  $|z|=2$ .

5. (a) If a series of complex numbers converges, then write to which the  $n$ th term converges as  $n$  tends to infinity. 1
- (b) Find the limit to which the sequence  $z_n = \frac{1}{n^3} + i$ ,  $n = 1, 2, \dots$  converges. 2
- (c) State and prove Liouville's theorem. 7

Or

Find the Taylor's series for the function  $\frac{1}{(1+z^2)(z+2)}$ , when  $|z| < 1$ .

6. (a) Define absolute convergence of a power series. 2
- (b) Define the circle of convergence of a power series. 2
- (c) Write when a power series is called uniformly convergent. 1
- (d) Find Laurent's series for the function

$$f(z) = \frac{4z+3}{z(z-3)(z+2)}$$

when  $2 < |z| < 3$ . 5

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