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6 SEM TDC DSE MTH (CBCS) 1 (H)

2024

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

(Hydromechanics)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the relation between material, local and convective derivatives. 1

(b) Fill in the blank : 1

If ϕ be the velocity potential, then it satisfies _____ equation.

(c) Write the difference between stream-lines and path lines. 3

(d) Determine the acceleration of a fluid particle from the flow field

$$\vec{q} = (Axy^2t)\hat{i} + (Bx^2yt)\hat{j} + (Cxyz)\hat{k}$$ 3

(e) Deduce the equation of continuity in Cartesian coordinates. 7

Or

If $u = \frac{ax - by}{x^2 + y^2}$, $v = \frac{ay + bx}{x^2 + y^2}$ and $w = 0$, investigate the nature of the liquid motion.

2. (a) What is called conservative field of force? 1

(b) Write down the Bernoulli's equation for steady and irrotational flow. 1

(c) State and prove Kelvin's circulation theorem. 5

(d) A stream in a horizontal pipe after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth $\frac{s^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$ below the pipe, s being the delivery per second. 5

Or

Deduce the equation of motion for impulsive force.

3. (a) Define acyclic irrotational motion. 1

(b) Choose the correct answer : 1

Let C be the closed curve and Γ be the circulation. Then

(i) $\Gamma = \int_C \vec{q} \cdot d\vec{r}$

(ii) $\Gamma = \int_C \vec{q} \times d\vec{r}$

(iii) $\Gamma = \int_C |\vec{q}|\, d\vec{r}$

(iv) None of the above

(c) Show that kinetic energy of liquid is given by

$$-\frac{1}{2}\rho \int_S \phi \frac{\partial \phi}{\partial n} dS$$

where ϕ is the single-valued velocity potential over the surface S, ρ be the density of the fluid. 6

Or

Prove that there cannot be two different forms of irrotational motion for a given confined mass of incompressible inviscid liquid whose boundaries are subject to the impulses.

4. *(a)* Define density of a homogeneous substance. 1

(b) Write True or False : 1

In a fluid, at rest under gravity, the pressure is the same at all points in the same horizontal plane.

(c) Fill in the blank : 1

The rate of increase of the pressure in any direction is equal to the product of the _____ and the component of external forces in that direction.

(d) Find the necessary condition that must be satisfied by a given system of external forces, so that the fluid may maintain equilibrium. 3

(e) Prove that pressure at a point of a fluid at rest is same in all directions. 6

(f) Show that the specific gravity of a mixture of n liquids is greater when equal volumes are taken than when equal weights are taken, assuming no change in volume as the result of mixing. 6

Or

State and prove the necessary and sufficient condition that a given distribution of forces (X, Y, Z) can keep a liquid in equilibrium.

5. *(a)* Write True or False : 1

The principle of Archimedes is the result to find the resultant thrust on a solid immersed in a fluid.

(b) Fill in the blank : 1

The whole pressure of a heavy homogeneous liquid on a plane is equal to the product of the area and the pressure at its _____.

(c) Prove that the position of center of pressure of a plane area is independent of the inclination of the area to the vertical. 6

(d) A circular area of radius a is immersed with its plane vertical and center at a depth h. Find the depth of center of pressure. 7

Or

A solid hemisphere of radius a is placed with its center at a distance h below the surface of water and has its plane face vertical. Find the horizontal thrust on the curved surface. Find also the resultant thrust on it.

6. *(a)* State the condition of equilibrium of a body freely floating in a liquid. 2

(b) A thin rod of weight W is loaded at one end with P of insignificant volume. If the rod floats in an inclined position with $\frac{1}{n}$th of its length out of the water, then prove that $(n-1)P = W$. 5

(c) Show that the equilibrium is stable, unstable, or neutral according as the metacenter is above, below or on the center of gravity of the body. 5

Or

A uniform rod of length $2l$ can turn freely about one end which is fixed at height $h(<2l)$ above the surface of the liquid. If the densities of the rod and the liquid be ρ and σ respectively, show that the rod can rest either in a vertical position or inclined at an angle θ to the vertical such that

$$\cos\theta = \frac{h}{2l}\sqrt{\frac{\sigma}{\sigma-\rho}}$$

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