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5 SEM TDC MTMH (CBCS) C 11

2024

(November)

MATHEMATICS

(Core)

Paper : C–11

(Multivariate Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* State the range of

$$f(x, y) = \frac{1}{xy}$$ 1

(b) Fill in the blank : 1

The definition of limit of $f(x, y)$ applies to the boundary points and _____ of the domain of f.

(c) State when

$$\lim_{(x, y)\to(x_0, y_0)} f(x, y)$$

does not exist. 1

(d) Investigate the existence of the limit

$$\lim_{(x, y)\to(0, 0)} \frac{x^2 + y}{y}$$ 2

(e) Find f_x and f_y, given $f(x, y) = \log_y x$. 2

(f) Use chain rule to find $\frac{dw}{dt}$ where $w = x^2 + y^2$ along the path $x = \cos t$ and $y = \sin t$ at $t = \pi$. 2

(g) If $w = f(x, y)$ is differentiable and both x and y are differentiable of t, then show that w is a differentiable function of t and

$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$ 3

Or

If $w = yz + zx + xy$ and $x = r + s$, $y = r - s$, $z = rs$, then find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ at $(r, s) = (2, 1)$.

(h) Determine ∇f at $(1, 2, -2)$ where

$$f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}} + \log xyz$$ 4

(i) Find the directional derivative of

$$f(x, y, z) = 3e^x \cos yz$$

at the origin in the direction of

$$2\hat{i} + \hat{j} - 2\hat{k}$$ 4

Or

Find the tangent plane and normal to the surface $x + y + z = 1$ at $(0, 1, 0)$.

(j) Find the local extrema or saddle point as applicable of the function

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$$ 5

(k) Use the method of Lagrange's multiplier to maximise $f(x, y) = xy$ subject to the constraint $x + y = 16$. 5

Or

Find the points on the surface $z^2 = xy + 4$ closest to the origin.

2. *(a)* Find the y-limits of integration for the integral

$$\iint_R f(x, y)\,dA$$

where R is the region bounded by the line $x+y=1$ and a circle of radius 1 with its centre at the origin. 1

(b) Sketch the region of integration on the plain paper for the integral

$$\int_0^3 \int_0^2 f(x, y)\,dx\,dy$$ 1

(c) Write any one iterated integral for the double integral

$$\iint_R f(x, y)\,dA$$

where R is the triangular region with the vertices (0, 0), (1, 0) and (0, 1). 2

(d) Reverse the order of integration

$$\int_0^1 \int_2^{4-2x} dy\,dx$$ 2

(e) Find the volume of the region between the cylinder $z=y^2$ and xy-plane bounded by the planes $x=0$, $x=1$, $y=-1$, $y=1$. 4

(f) Set up the iterated integral for evaluating the integral

$$\iiint_D f(r, \theta, z)\,dz\,r\,dr\,d\theta$$

over the region D which is a prism with its base on the xy-plane bounded by the x-axis and the lines $y=x$ and $x=1$ and whose top lies in the plane $z=2-y$. 5

(g) Write any two different iterated triple integrals for determining the volume of the tetrahedron bounded by the coordinate planes and the plane

$$x+\frac{y}{2}+\frac{z}{3}=1$$ 2½+2½=5

3. *(a)* Define Jacobian of the transformation

$$x=f(u, v);\; y=g(u, v)$$ 1

(b) If

$$u=\frac{1}{2}(x+1),\; v=\frac{1}{3}(y+4);\; w=2z+4$$

find $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$. 3

(c) Evaluate $\int_C (xy + y + z)\,ds$ along the curve

$$\bar{r}(t) = 2t\hat{i} + t\hat{j} + (2-2t)\hat{k};\ 0 \le t \le 1$$ 3

(d) Show that the work done $\int \bar{F} \cdot d\bar{r}$ around every closed loop in an open region D is zero if and only if $\bar{F}$ is conservative in D. 4

(e) State the fundamental theorem on line integrals. If $\bar{F}$ is a vector field whose components are continuous throughout an open connected region D in space, then there exists a differentiable function $f(x, y, z)$ such that $\bar{F} = \nabla f$, show that

$$\int_A^B \bar{F} \cdot d\bar{r}$$

is path independent where

$$\bar{r} = g(t)\hat{i} + h(t)\hat{j} + k(t)\hat{k};\ a \le t \le b$$

is a smooth curve joining A and B in D. 1+3=4

4. (a) Find the curl of the function

$$\bar{F}(x, y) = (x^2 - y)\hat{i} + (xy - y^2)\hat{j}$$ 1

(b) State Green's theorem in circulation-curl or tangential form. 2

(c) Integrate $f(x, y, z) = x + y + z$ over the surface of the cube cut from the first octant by the planes $x = a;\ y = a;\ z = a$. 3

(d) Evaluate $\iint_S f\,d\sigma$ where S is the surface area of the cone $z = \sqrt{x^2 + y^2};\ 0 \le z \le 1$ and $f(x, y, z) = x^2$. 4

(e) State and prove Stokes' theorem. 5

Or

State and prove divergence theorem.

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