3 SEM TDC PHYH (CBCS) C 5

2024

(Nov/Dec)

PHYSICS

(Core)

Paper : C–5

(Mathematical Physics—II)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The following differential equation

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0$$

is known as

(i) Legendre's equation

(ii) Bessel's equation

(iii) Laguerre's equation

(iv) Hermite's equation

(b) The value of $erf_c(x) + erf_c(-x)$ is

(i) 1 (ii) 2

(iii) -1 (iv) 0

(c) The value of $\Gamma(\frac{1}{2})$ is

(i) $\sqrt{\pi}$ (ii) $\frac{-\pi}{2}$

(iii) $-2\sqrt{\pi}$ (iv) 0

(d) The value of Legendre polynomial $P_1(x)$ is

(i) 1 (ii) x

(iii) $\frac{1}{2}$ (iv) None of the above

(e) The value of $P_n(-1)$ is

(i) 1 (ii) -1

(iii) 0 (iv) None of the above

2. (a) Write down the Parseval's formula for Fourier series. 2

(b) Expand the function $f(x)$ in a Fourier series where $-\pi < x < \pi$ and $f(x)$ is given by

$$f(x) = 0 \text{ when } -\pi < x \le 0$$
$$= \frac{\pi x}{4} \text{ when } 0 < x \le \pi$$

Hence show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

4+2=6

(c) Expand $f(x) = e^x$ in a cosine series over (0, 1). 3

3. (a) What do you mean by ordinary and singular points of a differential equation? Find the nature of the point $x = -1$ with reference to the differential equation

$$x^2(x+1)\frac{d^2y}{dx^2} + (x^2 - 1)\frac{dy}{dx} + 2y = 0$$

1+2=3

(b) Solve any *one* of the following using Frobenius method : 5

(i) $9x(1-x)y'' - 12y' + 4y = 0$

(ii) $xy'' + y + xy = 0$

(c) Prove that

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2}{2n-1}\delta_{mn}$$

3

Or

Express $2 - 3x + 4x^2$ in terms of Legendre polynomials.

(d) Prove the following : 2×2=4

(i) $P_n(1) = 1$

(ii) $P_n'(1) = \frac{1}{2}n(n+1)$

4. Show that

$$\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$$

3

Or

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

5. Answer any *two* of the following : 3×2=6

(a) Define absolute error, relative error and percentile error. 3

(b) $R = \frac{4xy^2}{z^3}$ and errors in x, y, z be 0·001, show that the maximum relative error at $x = y = z = 1$ is 0·006. 3

(c) State and prove the normal law of errors. 3

6. (a) Solve any *two* of the following partial differential equations by the method of separation of variables : 4×2=8

(i) $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$

(ii) $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ under the following conditions :

$$u = 0 \text{ at } t = 0; \ \frac{\partial u}{\partial t} = 0 \text{ at } x = 0$$

(iii) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ under the following conditions :

$$\text{At } x = 0, \ u = 0 \text{ and } \frac{\partial u}{\partial x} = 1 + e^{-3y}$$

(b) Find the solution of one-dimensional wave equation in Cartesian coordinates. 5

Or

Find the solution of 2-D Laplace's equation in spherical coordiantes.

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