1 SEM TDC PHYH (CBCS) C 1

2024

(November)

PHYSICS

(Core)

Paper : C–1

(Mathematical Physics—I)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+3y=0$ are respectively

(i) 2 and 2

(ii) 2 and 1

(iii) 1 and 2

(iv) 3 and 2

(b) The condition for differential equation of the form $M\,dx + N\,dy = 0$ to be exact is

(i) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

(iv) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(c) The order of a differential equation is always

(i) positive integer

(ii) negative integer

(iii) rational number

(iv) whole number

(d) The divergence of curl of a vector is

(i) 1

(ii) 0

(iii) $\frac{1}{2}$

(iv) $\frac{\pi}{2}$

(e) By Gauss divergence theorem $\int_V \vec{\nabla} \cdot \vec{A} dV$ is equal to

(i) $\int_S \vec{A} \cdot d\vec{S}$

(ii) $\int_C \vec{A} \cdot d\vec{S}$

(iii) $\int_C \vec{A} \cdot d\vec{r}$

(iv) $\int_V \vec{A} \cdot d\vec{V}$

2. Answer the following questions : 2×5=10

(a) Show that $|x|$ is continuous but not differentiable.

(b) For what values of a, $\vec{A}$ and $\vec{B}$ are perpendicular, if $\vec{A} = a\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2a\hat{i} + a\hat{j} - 4\hat{k}$?

(c) What is a Wronskian? How is it used to find the linear dependence of two functions?

(d) Show that $\vec{B}$ is perpendicular to $\vec{A}$ if $|\vec{B}| \neq 0$ and $\vec{B} = \frac{d\vec{A}}{dt}$.

(e) Evaluate using the property of Dirac delta function $\int_{-\infty}^{\infty} x\delta(x-4)\,dx$.

3. Answer any *five* of the following questions : 4×5=20

(a) What do you mean by linearly dependent and linearly independent solutions of a homogeneous equation? If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are two solutions of $y'' + 9y = 0$, then show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions. 1+3=4

(b) If $z(x+y) = x^2 + y^2$, then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$ 4

(c) Solve the following differential equations : 2×2=4

(i) $\dfrac{d^2 z}{dx^2} = \cos(2x + 3y)$

(ii) $(2x \log x - xy)dy + 2y\,dx = 0$

(d) What is directional derivative? Find the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at $(1, 1, -1)$ in the direction $2\hat{i} + \hat{j} - \hat{k}$. 1+3=4

(e) State Green's theorem in a plane. Starting from Green's theorem, show that the area bounded by a closed curve is given by $\frac{1}{2}\oint_C (x\,dy - y\,dx)$. 1+3=4

(f) State Bayes' theorem of probability. 6 cards are drawn from a pack of 52 cards. What is the probability that 3 will be red and 3 black? 1+3=4

4. Answer any *three* of the following questions : 6×3=18

(a) What are complementary function and particular integral of a differential equation? Solve the differential equation

$$\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2$$

if $y(0) = 0$ and $y'(0) = \frac{1}{2}$. 1+5=6

(b) Evaluate

$$\iint \vec{F} \cdot \hat{n}\, dS$$

where

$$\vec{F} = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$$

and S being the surface of the sphere having centre $(3, -1, 2)$ and radius 3. 6

(c) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Also find the work done in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$. 2+2+2=6

(d) What are curvilinear coordinates? Describe the term 'scale factor' in curvilinear coordinates. Derive the expression for divergence of a vector in curvilinear coordinates. Hence write its expression in spherical polar coordinates. 1+2+3=6

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