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**5 SEM TDC MTMH (CBCS) C 11**

**2025**

( Nov/Dec )

**MATHEMATICS**

( Core )

Paper : C-11

**( Multivariate Calculus )**

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) State the domain of the function

$$f(x, y) = \sqrt{4 - x^2 - y^2} \quad 1$$

- (b) Find  $\left(\frac{\partial f}{\partial y}\right)_{(1,0)}$  where

$$f(x, y) = x^3 y^2 + e^{xy^2} \quad 2$$

( 2 )

(c) Show that

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{2xy}{x^2 + y^2}$$

does not exist.

3

(d) Using definition, show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}; & (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

is continuous at the origin.

3

Or

For the function

$$f(x, y) = \begin{cases} \frac{x^3 + y^3}{x - y}; & x \neq y \\ 0 & ; x = y \end{cases}$$

show that  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$  exist at (0, 0) and are equal.

(e) Show that  $f_{xy} = f_{yx}$  where

$$f(x, y) = x \sin y + y \sin x + xy$$

3

(f) Find the values of  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at the point  $(\pi, \pi, \pi)$  for the function

$$\sin(x + y) + \sin(y + z) + \sin(z + x) = 0$$

4

( 3 )

Or

Find  $\left(\frac{dw}{dt}\right)_{t=1}$  where  $w = z - \sin xy$  and  $x = t$ ,  $y = \log t$ ,  $z = e^{t-1}$ .

(g) Find an equation for tangent plane to the surface  $z = \tan^{-1} \frac{y}{x}$  at the point  $(1, \sqrt{3}, \frac{\pi}{3})$ .

4

Or

Find the directional derivative of  $f(x, y) = e^{x^2 y^2}$  at  $P_0(1, -1)$  in the direction towards  $Q(2, 3)$ .

(h) Find a vector that is normal to the level surface  $x^2 + 2xy - yz + 3z^2 = 7$  at the point  $(1, 1, -1)$ .

2

(i) Determine the stationary points of the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$  and classify the points.

4

Or

Find the maximum and minimum values of the function  $f(x, y) = 3x + 4y$  on the circle  $x^2 + y^2 = 1$ .

( 4 )

(i) Find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$  where

$$\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz) \quad 4$$

Or

If  $\vec{F} = x^2y\hat{i} + xz\hat{j} + 2yz\hat{k}$ , find  $\nabla \cdot (\nabla \times \vec{F})$ .

2. (a) Evaluate

$$\iint_R x^2 y^5 dA$$

where  $R: [1, 2; 0, 1]$ . 2

(b) Show that

$$\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dx \quad 4$$

3. (a) Define Jacobian of a function of two variables. 1

(b) Evaluate

$$\iint_E \sin\left(\frac{x-y}{x+y}\right) dx dy$$

where  $E$  is the region bounded by the coordinate axes and  $x+y=1$  in the first quadrant. 4

( 5 )

Or

Use transformations  $u = x - y$  and  $v = 2x + y$  to evaluate the integral

$$\iint_R (2x^2 - xy - y^2) dx dy$$

where  $R$  is the region bounded by the lines  $y = -2x + 4$ ,  $y = -2x + 7$ ,  $y = x - 2$  and  $y = x + 1$ .

(c) Define flux across a plane curve. 1

(d) Find the value of

$$\int_C \{(x+y)^2 dx + (x^2 - y) dy\}$$

taken in the clockwise sense along the closed curve  $C$  formed by  $y^3 = x^2$  and the chord joining  $(0, 0)$  and  $(1, 1)$ . 4

Or

Evaluate  $\int_C f(x, y) dS$ , where  $f(x, y) = x + y$  and  $C$  is the circle  $x^2 + y^2 = 4$  in the first quadrant from  $(2, 0)$  to  $(0, 2)$ .

(e) State and prove the fundamental theorem of line integrals. 5

( 6 )

Or

Find the value of  $\int_C (x^2 y dx + xy^2 dy)$  taken in the clockwise sense along the hexagon whose vertices are  $(\pm 3a, 0)$   $(\pm 2a, \pm \sqrt{3}a)$ .

4. (a) State and prove Green's theorem. 5

Or

Prove that the line integral

$$\int_C \frac{x dy - y dx}{x^2 + y^2}$$

taken in the positive direction over any closed contour with the origin inside it, is equal to  $2\pi$ .

- (b) Write one property of surface integral. 1  
(c) Evaluate

$$\iint_S x dy dz + dz dx + xz^2 dx dy$$

where  $S$  is the outer side of the part of the sphere  $x^2 + y^2 + z^2 = 1$  in the first octant. 4

Or

Use Stokes' theorem to find the line integral  $\int_C x^2 y^3 dx + dy + z dz$ .

( 7 )

- (d) Prove that the flux of a vector field  $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$  where  $M, N, P$  are functions of  $x, y$  and  $z$  across a closed piecewise smooth oriented surface  $S$ , in the direction of its outward unit normal field  $\hat{n}$  is equal to

$$\iiint_D \nabla \cdot \vec{F} dV$$

where  $D$  is the convex region without holes or bubbles. 5

Or

Use divergence theorem to find the outward flux of  $\vec{F}$  across the boundary of the region  $D$ , where

$$\vec{F} = (y-x)\hat{i} + (z-y)\hat{j} + (y-x)\hat{k}$$

and  $D$  is the cube bounded by the planes  $x = \pm 1, y = \pm 1$  and  $z = \pm 1$ .

Or

Evaluate  $\iint_D \frac{dA}{y^2 + 1}$ ;  $D$  is the triangle bounded by  $x = 2y, y = -x$  and  $y = 2$ .

- (e) Reverse the order of integration in the iterated integral

$$\int_0^2 \int_1^{e^x} f(x, y) dy dx$$

2

- (f) Using double integral, find the area of the region between  $y = \cos x$  and  $y = \sin x$  over the interval  $0 \leq x \leq \frac{\pi}{4}$ . 2
- (g) Evaluate  $\iiint_B z^2 y e^x dV$  where  $B$  is the box given by  $0 \leq x \leq 1$ ,  $1 \leq y \leq 2$ ,  $-1 \leq z \leq 1$ . 2
- (h) Compute the volume of the solid bounded by the sphere  $x^2 + y^2 + z^2 = 4$  and the surface of the paraboloid  $x^2 + y^2 = 3z$ . 4

Or

Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane  $x + \frac{y}{2} + \frac{z}{3} = 1$ .

- (i) Evaluate the following integral by changing the order of the integration in an appropriate way :

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4 \cos(x^2)}{2\sqrt{z}} dx dy dz \quad 4$$

Or

Evaluate :

$$\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_{\sec \phi}^2 3\rho^2 \sin \phi d\rho d\phi d\theta$$

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