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5 SEM TDC DSE MTH (CBCS)

1.1/1.2/1.3 (H)

2025

(Nov/Dec)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions.

Paper : DSE–1.1

(Analytical Geometry)

1. Answer the following questions :

(a) Write the vertex of the conic

$$(x-3)^2 = 2(y+4)$$ 1

(b) Find the equation of the ellipse whose ends of major axis are $(0, \pm 6)$ and passes through the point $(-3, 2)$. 4

(c) Identify and sketch the curve $x = y^2 - 4y + 2$, and also label the focus, vertex and directrix. 5

(d) Describe the graph of the curve

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0$$

Also find its centre and foci. 5

Or

Describe the graph of the hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0$$

and sketch its graph.

2. Answer the following questions :

(a) Fill in the blank : 1

The set of points in the plane, the sum of whose distances to two fixed points is a positive constant greater than the distance between the fixed points is _____.

(b) Write True or False : 1

A hyperbola is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

(c) Define a parabola. 1

(d) Find the equation of the ellipse one of whose foci is (–1, 1), eccentricity is $\frac{1}{2}$ and the corresponding directrix is $y = x + 3$. 6

(e) Find the equation of the parabola whose axis is parallel to the y-axis that has its vertex at (5, –2) and passes through the point (9, 5) and also sketch it. 6

Or

Find and sketch the curve of the hyperbola whose foci (6, 4) and (–4, –4) and eccentricity is 2.

3. Answer the following questions :

(a) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents hyperbola. 1

(b) Determine a rotation angle θ that will eliminate the xy-term of the conic

$$2x^2 + xy + 2y^2 + x - y = 0$$ 2

(c) Show that the graph of the equation

$$x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$$

is a hyperbola. Find its foci, vertices and asymptotes. 6

(d) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 45°$. Find an equation of the curve $3(x')^2 + (y')^2 = 6$ in xy-coordinate. 6

Or

Identify and sketch the curve

$$153x^2 - 129xy + 97y^2 - 30x - 40y - 200 = 0$$

4. Answer the following questions :

(a) Define sphere. 1

(b) Write True or False : 1

The section of a sphere by a plane is a sphere.

(c) Find the equation of the sphere whose centre is (1, 3, 2) and radius is 6 units. 4

(d) Find the equation of the sphere through the origin and intersecting coordinate axes at distances a, b, c from the origin. 5

Or

A plane passes through a fixed point (a, b, c) and meets the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

5. Answer the following questions :

(a) Define great circle. 1

(b) Write the condition that the plane

$$ax + by + cz + d = 0$$

be a tangent plane to the sphere

$$x^2 + y^2 + z^2 = r^2$$ 1

(c) Find the radius and centre of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,\ x - 2y + 2z = 3$$ 5

(d) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y + 4z = 8$$

is a great circle. 5

Or

Prove that the two spheres

$$x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$$
$$x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$$

touch each other.

6. Answer the following questions :

(a) Write the name of cylindrical surface given by the equation

$$\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{12}=1$$ 1

(b) Prove that the two spheres

$$x^2+y^2+z^2+6y+2z+8=0$$
$$x^2+y^2+z^2+6x+8y+4z+20=0$$

intersect each other orthogonally. 2

(c) Show that the plane $2x-2y+z+12=0$ touches the sphere

$$x^2+y^2+z^2-2x-4y+2z-3=0$$

Also find the point of contact. 4

(d) Classify and sketch the quadric surface (any *one*) : 5

(i) $36x^2+9y^2+16z^2=144$

(ii) $4x^2-3y^2+12z^2+12=0$

Paper : DSE–1.2

(Portfolio Optimization)

1. Answer any *five* of the following questions : 1×5=5

(a) Define investment.

(b) What is the relation between risk and return?

(c) Define risk.

(d) What is risk-free asset?

(e) What is diversification?

(f) Define portfolio.

2. *(a)* Find annual holding period return (annual HPR) and annual holding period yield (annual HPY) of an investment of ₹100 for six months that earned a return of ₹12. 4

(b) In an investment with perfect certainty, how many terms will be present in the expected return of the investment? Calculate the expected rate of return of the following economic scenario : 1+2=3

Economic Condition	*Probability*	*Rate of Return*
Strong economy	0·15	0·20
Weak economy	0·25	– 0·10
No major change in economy	0·60	0·10

(c) Define the variance for an expected rate of return of an investment. What will be the variance of an investment with perfect certainty of return. Define coefficient of variation of risk of an investment. 1+2+1=4

(d) Describe different types of risk of an investment. 5

(e) Define risk premium and systematic risk. 2+2=4

(f) Write a short note on Mutual fund. 5

Or

Describe investment objective and investment constraints.

3. (a) Mention three assumptions of Markowitz's portfolio theory. 3

(b) Find the variance and standard deviation of the following investment scenario : 4

Possible Rate of Return (R_i)	*Expected Security Return* $E(R_i)$	*Probabilities* (P_j)
0·08	0·105	0·35
0·10	0·105	0·30
0·12	0·105	0·20
0·14	0·105	0·15

(c) Describe variance and standard deviation of returns for a portfolio of investment. 5

Or

Calculate portfolio standard deviation of two assets with equal weights $w_1 = 0{\cdot}50$, $w_2 = 0{\cdot}50$ for two correlation coefficients $r_{1,2} = 1{\cdot}00$ and $r_{1,2} = 0{\cdot}50$ where $\sigma_1 = 0{\cdot}10$, $\sigma_2 = 0{\cdot}10$. Apply the formula of covariance $\text{cov}_{ij} = r_{ij}\sigma_i\sigma_j$.

(d) State and prove two-fund theorem. 5

(e) Define risk-free portfolio. 2

(f) Write short notes on any *two* of the following : 3×2=6

(i) Optimal portfolio

(ii) Assumptions of capital market theory

(iii) Portfolio with short sales

4. Answer any *three* of the following questions : 5×3=15

(a) Derive the risk-return combination equation of capital market theory.

(b) Describe capital market line.

(c) Write a short note about beta of an asset and a portfolio.

(d) Describe security market line (SML).

5. (a) Identify the following stocks which are properly valued, overvalued and undervalued : 5

Stock	Expected Return $E(R_i)$	Estimated Return
A	7·80	8·00
B	9·00	6·20
C	9·60	15·15
D	10·60	5·16
E	3·80	6·00

(b) Describe sharp portfolio performance measure with example. 5

Paper : DSE–1.3

(Financial Mathematics)

1. (a) Write the inverse supply function of the function $8p+5q=40$. 1

(b) For demand and supply functions, the equilibrium set may contain more than one point. (State True or False) 1

(c) Let (100, 8) be a point on the demand curve of a quantity. Write the price per unit at that point. 1

(d) Define an annuity. 1

(e) Let demand and supply functions are given by $q+2p=22$ and $10q-5p=70$. Find the equilibrium set. 2

(f) An amount ₹2,000 is invested at the rate of interest 10% per annum. Find the capital growth after one year compounded half yearly. 4

Or

Let the supply and demand functions for an item are $q=12p-4$ and $q=8-4p$ respectively. Find the selling price and quantity sold after introduction of excise tax T.

2. Answer any *two* of the following questions : 4×2=8

(a) Let a market is described by $q^S = 2p - 3$, $q^D = 18 - p$. Determine whether the Cobweb model predicts stable or unstable equilibrium.

(b) Describe the economic interpretation of the model described by supply and demand functions

$$S = \{(q, p) : q = bp - a\}$$
$$D = \{(q, p) : q = c - dp\}$$

respectively.

(c) Let the demand and supply sets are

$$D = \{(q, p) : q + p = 24\}$$
$$S = \left\{(q, p) : q + 9 = \frac{p}{2}\right\}$$

Find the price sequence, where the initial price $p_0 = 23$.

3. *(a)* Let $f(x)$ be a continuous function in the interval $[a, b]$. Write the condition when $f(x)$ will be maximum at a point $c \in (a, b)$. 2

(b) Define critical point of a function. 2

(c) Answer any *one* of the following : 4

(i) A firm has cost function

$$C(q) = 2x^3 + 3x^2 + 6x + 10$$

Show that its marginal cost is always positive.

(ii) Let supply and demand sets for an item are given by $q = 6p - 2$ and $q = 5 - 3p$ respectively. An excise tax T is imposed. Find the value of T at which excise tax is maximum.

4. *(a)* Write how elasticity of demand is related to revenue as price increases. 1

(b) If revenue $R = qp$, where q is the quantity and p is the price, then write the value of $\frac{dR}{dp}$. 1

(c) Elasticity of demand is the ratio of two things. Write that two things. 2

(d) Answer any *two* of the following questions : 4×2=8

(i) Show that the optimum production level occurs when marginal revenue equals marginal cost.

(ii) Let $C(q) = 50 + 7q - 2q^2 + q^3$ be the cost function of an efficient small firm. Find its profit function.

(iii) For the cost function

$$C(q) = 100 + 20q - 6q^2 + q^3$$

of an efficient small firm, determine—

(1) their fixed cost;

(2) their startup point.

5. Let

$$f(x, y) = xe^y + x^2y$$

Find $\frac{\partial f}{\partial y}$. 1

(b) Let $R(x, y) = x + 2y$ be the revenue function and $C(x, y) = 2 + x^3 + 2xy + y$ be the cost function. Find its profit function. 2

(c) Find the critical points of the function

$$f(x, y) = x^4 + 2x^2y + 2y^2 + y$$ 3

(d) If $f(x, y) = x^3 - y^3 - 2xy + 1$, classify the critical points of f. 4

Or

Find the maximum value of the function

$$f(x, y) = 6 + 4x - 3x^2 + 4y + 2xy - 3y^2$$

6. (a) Define a return matrix. 1

(b) Write the name of the portfolio which costs nothing, cannot lose and in at least one state yields a profit. 1

(c) Answer any *two* of the following questions : 5×2=10

(i) Let $y = [500 \quad -1000 \quad 600]$ be the portfolio and

$$R = \begin{bmatrix} 1{\cdot}2 & 0{\cdot}95 \\ 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}9 & 1{\cdot}1 \end{bmatrix}$$

be the return matrix. Compute the values of the portfolio in various states.

(ii) Describe a two-industry economy.

(iii) Let an input-output model with two industries, for which matrix coefficients is

$$A = \begin{bmatrix} 0{\cdot}3 & 0{\cdot}1 \\ 0{\cdot}2 & 0{\cdot}4 \end{bmatrix}$$

Determine the production schedule $\bar{x}$ in terms of external demand $\bar{d}$.

7. *(a)* Define a cash flow stream. 1

(b) Write the meaning of the cash flow stream (−15, 20). 1

(c) Write one purpose of insurance. 1

(d) Write which have to be accepted to get greater expected return. 1

(e) Write about arbitrage. 2

8. *(a)* Write the type of capital growth under compound interest. 1

(b) Write the total value of an amount A invested for n years at a simple interest r. 1

(c) Define internal rate of return. 2

(d) Answer any *two* of the following questions : 5×2=10

(i) Describe the effect of compounding at various intervals.

(ii) Describe different types of bonds.

(iii) Find internal rate of return of the cash flow (−2, 0, 1, 1).

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