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5 SEM TDC DSE MTH (CBCS)
2.1/2.2/2.3/2.4 (H)

2025

(Nov/Dec)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–2.1/2.2/2.3/2.4

The figures in the margin indicate full marks for the questions

Paper : DSE–2.1

(MATHEMATICAL MODELLING)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer the following as directed : 1×5=5

(a) Define ordinary point.

(b) $\Gamma(1) =$ _____ (Fill in the blank)

(c) Define Bessel's equation.

(d) Find $L\{1\}$.

(e) State a sufficient condition for the existence of Laplace transform of a function.

2. Answer any *one* of the following : 2

(a) Determine whether $x = 0$ is an ordinary point or a regular singular point of the differential equation

$$x^2 \frac{d^2y}{dx^2} + 7x(x+1)\frac{dy}{dx} - 3y = 0$$

(b) Find $L^{-1}\left(\frac{1}{s}\right)$.

3. Express $4x^2 - 3x + 2$ in terms of Legendre polynomial. 3

4. Find Laplace transform (any *two*) : 2½×2=5

(a) $(t^2+1)^2$

(b) $\cosh at$

(c) $\frac{1}{\sqrt{\pi t}}$

5. Find the solution in series of

$$(x^2+1)y'' + xy' - xy = 0$$

about $x = 0$. 5

6. Show that

$$L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\left(\frac{\pi}{s}\right)}\, e^{-\frac{1}{4}s}$$ 5

Or

Show that the Laplace transform of the function $F(t) = t^n$, $-1 < n < 0$ exists, although it is not a function of class *A*.

7. Answer the following :

(a) What is feasible region? 1

(b) When is an optimization problem said to be a 'linear program'? 2

(c) Define dynamic program and stochastic program. 2

(d) Write the Monte Carlo inventory algorithm. 4

(e) "Monte Carlo simulation is a deterministic process." Write true or false.

Using Monte Carlo simulation, write an algorithm to calculate the area under the curve $f(x) = \sqrt{x}$, over the interval

$$\frac{1}{2} \le x \le \frac{3}{2}$$ 1+3=4

8. Answer any *two* of the following : 5×2=10

(a) A carpenter realizes a net unit profit of ₹ 25 per table and ₹ 30 per bookcase. He has up to 690 board-feet of lumber and up to 120 hours of labour to devote weekly to the project. He estimates that it requires 20 board-feet of lumber and 5 hours of labour to complete a table, and [illegible] board-feet of lumber and 4 hours of labour to complete a bookcase. Determine how many tables and how many bookcases he should make each week to maximize his profit.

(b) Solve the following linear program algebraically :

$$\begin{aligned} \text{Maximize } & 10x + 35y \\ \text{subject to } & 8x + 6y \le 48 \\ & 4x + y \le 20 \\ & y \ge 5 \\ & x,\ y \ge 0 \end{aligned}$$

(c) What is dichotomous search method? Write dichotomous search algorithm to maximize a function $f(x)$ over the interval $[a, b]$.

9. Answer any *two* of the following : 6×2=12

(a) Explain linear congruence and use it to generate 10 random numbers using $a = 1$, $b = 7$ and $c = 10$.

(b) Solve using simplex method :

$$\begin{aligned} \text{Minimize } & 6x + 5y \\ \text{subject to } & x + y \ge 6 \\ & 2x + y \ge 9 \\ & x,\ y \ge 0 \end{aligned}$$

(c) Use dichotomous search method to maximize $f(x) = -x^2 - 2x$ over the interval $-3 \le x \le 6$ with a tolerance of $t = 0 \cdot 2$ and $\varepsilon = 0 \cdot 01$.

Paper : DSE–2.2

(**MECHANICS**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. *(a)* Determine the moment about the origin O of the force

$$\vec{F} = -7N\hat{i} - 2N\hat{j} + 3N\hat{k}$$

which acts at a point A. The position vectors of A are *(i)* $\vec{r} = 4m\hat{i} + 2m\hat{j} - m\hat{k}$ and *(ii)* $\vec{r} = -8m\hat{i} + 3m\hat{j} + 4m\hat{k}$. 4

(b) What do you mean by distributed force system? Give an example of it. 2+1=3

(c) Show that two couples in the same plane whose moments are equal and of the same sign are equivalent to one another. 6

(d) A 700 N circus performer causes a 0·15 m sag in the middle of 12 m tight rope with a 5000 N initial tension. What additional tension is induced in the cable? What is the cable tension when the performer is 3 m from the end and the sag is 0·12 m? 7

Or

Show that the necessary condition for a rigid body to be in equilibrium is that the resultant force and resultant couple point for any point to be zero vectors.

2. *(a)* Write down the Coulomb's laws of friction. 2

(b) The block of weight W is to be moved up an inclined plane. A rod of length C with negligible weight is attached to the block and the force $\vec{F}$ is applied to the top of the rod. If the coefficient of static friction is μ_s, determine the maximum length C for which the block will begin to slide rather than tip in terms of d (block's breadth) and μ_s. 5

(c) Find I_{xx}, I_{yy} and I_{xy} for the area bounded by $y = e^x$ and $y = -e^x$. 6

Or

Find the first moments of the area bounded by $y^2 = 25x$ and x-axis and $y = 10$ about x- and y-axes.

(d) Establish the relation between second moments and product of inertia. 5

(e) Define centroid. Find the centroid of the area bounded by $y^2 = 4x$ and $x^2 = 4y$. 2+5=7

3. (a) What do you mean by conservative force field? Show that in a conservative force field, $\vec{F} = -\nabla V$, where the symbols have their usual meanings. 2+3=5

(b) For a given conservative force field

$$\vec{F} = (5z\sin x + y)\hat{i} + (4yz + x)\hat{j} + (2y^2 - 5\cos x)\hat{k}$$

find the force potential. What is the work done on a particle starting at the origin and moving in a circular path of radius 2 to form a semicircle along the positive x-axis? 6

(c) Derive the moment of momentum equation for a system of particle. 6

(d) Establish the relation between acceleration vectors of a particle for two systems of references moving arbitrarily relative to each other. 6

Or

Find the kinetic energy of a rigid body rotating about a fixed point.

(e) Show that the sum of the potential energy and the kinetic energy for a particle remains constant for all times during the motion of a particle.

A particle is dropped with zero initial velocity down a frictionless chub. What is the magnitude of the velocity if the vertical drop during the motion is h ft? 6

(f) State and prove Chasles' theorem. 6

Paper : DSE–2.3

(NUMBER THEORY)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. *(a)* State Goldbach conjecture. 1

   (b) Prove that if $a \equiv b \pmod{n}$ and $m|n$, then $a \equiv b \pmod{m}$. 2

   (c) Using Fermat's theorem, find the remainder when $2 \times 26!$ is divided by 29. 3

   (d) Solve the following simultaneous congruences : 4

$$x \equiv 5 \pmod{6}$$
$$x \equiv 4 \pmod{11}$$
$$x \equiv 3 \pmod{17}$$

2. Answer any *two* of the following : 5×2=10

   (a) Determine all solutions in the positive integers of the Diophantine equation

$$18x + 5y = 48$$

   (b) Prove that $53^{103} + 103^{53}$ is divisible by 39.

   (c) State and prove Wilson's theorem.

3. *(a)* Write the value of $\sigma(p)$, where p is a prime. 1

   (b) Write a reduced set of residues modulo 12. 1

   (c) For any positive integer n, show that 2

$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$$

   (d) Verify that 1000! terminates in 249 zeros. 3

   (e) Prove that if n is an odd integer, then $\phi(2n) = \phi(n)$. 2

   (f) What is a multiplicative function? Prove that if f is a multiplicative function and $f(x) \neq 0$ for some $n \in \mathbb{N}$, then $f(1) = 1$. 1+2=3

   (g) Define Dirichlet product of two functions and prove that it is commutative. 1+2=3

   (h) For any integer $n > 1$, prove that 3

$$n^{\frac{\tau(n)}{2}} = \prod_{d|n} d$$

4. Answer any *three* of the following : 4×3=12

   (a) If $n \geq 1$ and $\gcd(a, n) = 1$, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.

   (b) For each positive integer $n \geq 1$, prove that $n = \sum_{d|n} \phi(d)$.

(c) If f is a multiplicative function and

$$f(n) = \sum_{d|n} F(d)$$

then by using Möbius inversion formula prove that F is also multiplicative.

(d) Use Euler's theorem to establish that 51 divides $10^{32n+9} - 7$.

5. (a) Find the order of 2 modulo 17. 1

(b) Define primitive Pythagorean triple. Find three different Pythagorean triples, not necessarily primitive, of the form 16, y, z. 1+2=3

Or

Prove that if p is an odd prime, then p^2 has a primitive root.

(c) Prove that if $F_n = 2^{2^n} + 1$, $n > 1$, is a prime, then 2 is not a primitive root of F_n. 4

Or

If $\gcd(m, n) = 1$, where $m > 2$ and $n > 2$, then show that the integer mn has no primitive roots.

(d) Solve the following quadratic congruence : 4

$$3x^2 + 9x + 7 \equiv 0 \pmod{13}$$

(e) Show that 7 and 18 are the only incongruent solutions of

$$x^2 \equiv -1 \pmod{5^2}$$ 4

(f) Define the Legendre symbol (a / p), where p is a prime and $(a, p) = 1$. Prove that if $a \equiv b \pmod{p}$, then $(a / p) = (b / p)$. 4

6. Answer any *two* of the following : 5×2=10

(a) Let the integer a has order k modulo n. Then show that $a^h \equiv 1 \pmod{n}$ if and only if $k | h$.

(b) Prove that if n has a primitive root, then it has exactly $\phi(\phi(n))$ of them.

(c) Prove that if p is a prime number and $d | p-1$, then the congruence $x^d - 1 \equiv 0 \pmod{p}$ has exactly d solutions.

Paper : DSE–2.4

(BIOMATHEMATICS)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

UNIT—I

1. Answer any *two* of the following : 7½×2=15

(a) A population is originally 100 individuals, but because of the combined effects of births and deaths, it triples each hour.

(i) Make a table of population size for $t = 0$ to 5, where t is measured in hours.

(ii) Give two equations modelling the population growth by first expressing P_{t+1} in terms of P_t and then expressing ΔP in terms of P_t.

(iii) What can you say about the birth-rate and death rate for this population?

(b) In the early stages of the development of a frog embryo, cell division occurs at a fairly regular rate. Suppose you observe that all cells divide, and hence the

number of cells double, roughly every half-hour.

(i) Write down an equation modelling this situation. You should specify how much real-world time is required by an increment of 1 in t and what the initial number of cells is.

(ii) Produce a table and graph of the number of cells as a function of t.

(c) Obtain a simple prey-predator model explaining in detail the assumptions taken. Also find the equilibrium positions.

UNIT—II

2. Answer any *two* of the following : 7½×2=15

(a) Consider the SI epidemic model. If the contact rate is 0·001 and the number of susceptible is 2000 initially, determine—

(i) the number of susceptible left after 3 weeks;

(ii) the density of susceptible when the rate of appearance of new cases is a maximum;

(iii) the time (in weeks) at which the rate of appearance of new cases is a maximum;

(iv) the maximum rate of appearance of new cases.

(b) In an SIS model, if the infection is spread only by a constant number of carriers, then show that

$$I(t) = \left(I_0 - \frac{\alpha CN}{\alpha C + \beta} \right) e^{[-(\alpha C + \beta)t]} + \frac{\alpha CN}{\alpha C + \beta}$$

where I and C are the numbers of infectives and carriers; N is total population; α and β are contact rate and susceptible rate respectively; I_0 is the infectives at $t = 0$.

(c) Let x and y respectively denote the proportion of susceptibles and carriers in a population. Suppose the carriers are identified and removed from the population at a rate β, so that $\frac{dy}{dt} = \beta y$. Suppose also that the disease spreads at a rate proportional to the product of x and y, thus

$$\frac{dx}{dt} = -\alpha xy$$

(i) Determine the proportions of carriers at any time t, where $y(0) = y_0$.

(ii) Use (i) to find the susceptibles at time t, where $x(0) = x_0$.

(iii) Find the proportion of population that escapes the epidemic.

UNIT—III

3. Answer any *two* of the following : 7½×2=15

(a) Consider the competition models for two species with populations N_1 and N_2

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - b_{12} \frac{N_2}{K_1} \right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - b_{21} \frac{N_1}{K_2} \right)$$

where only one species N_1 has limited carrying capacity. Investigate their stability and sketch the phase plane trajectories.

[Here, K_1, K_2 are carrying capacities; r_1, r_2 are linear birthrates of the populations N_1 and N_2 respectively; b_{12}, b_{21} measure the competitive effect of N_2 on N_1 and N_1 on N_2 respectively.] 4+3½=7½

(b) What is Routh-Hurwitz criteria? Explain with reference to multiple species communities. 2+5½=7½

(c) Discuss bifurcation and limit cycle with respect to any biological model.

UNIT—IV

4. Answer any *two* of the following : 7½×2=15

(a) Write a short note on any *one* of the following :

(i) One-species model with diffusion

(ii) Two-species model with diffusion

(b) For a blood vessel of constant radius R, length L and driving force $P = p_1 - p_2$, show that the average velocity of the flow is equal to half of the maximum velocity and the resistance is proportional to $\frac{L}{R^4}$.

(c) Consider arterial blood viscosity $\mu = 0 \cdot 027$ poise.

If the length of the artery is 2 cm, and radius 8×10^{-3} cm and $P = p_1 - p_2 = 4 \times 10^3$ dynes/cm^2, then find—

(i) $q_z(r)$ and the maximum peak velocity of blood;

(ii) the shear stress at the wall.

(Here, q_z denotes velocity along z-axis, p_1 and p_2 denote pressure at two ends of the artery.)

UNIT—V

5. Answer any *two* of the following : 10×2=20

(a) Let D & d and W & w respectively denote allele for tall & dwarf and round & wrinkled seeds of peas. Find the outcome of the product $DdWw \times ddWw$ using Punnett square or using probability. Also find the probability that the progeny of $DdWw \times ddWw$ is dwarf with round seeds. 6+4=10

(b) Explain in detail the Hardy-Weinberg equilibrium, mentioning the assumptions considered for the equilibrium.

(c) Compare and contrast the stage structure model with age structure model.

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