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5 SEM TDC PHYH (CBCS) C 11

2025

(Nov/Dec)

PHYSICS

(Core)

Paper : C–11

(Quantum Mechanics and Applications)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) Quantum mechanical operator of total energy is

(i) $\frac{\hbar}{i}\nabla$ (ii) $i\hbar\frac{\partial}{\partial t}$

(iii) $\frac{\hbar}{i}(r\times\nabla)$ (iv) $-\frac{\hbar^2}{2m}\nabla^2$

(b) The energy of a linear harmonic oscillator is

(i) discrete and equispaced

(ii) continuous

(iii) partly discrete and partly continuous

(iv) discrete but not equispaced

(c) If A is the operator, operating on a state function ψ, then the possible values of any physical quantity of a system in the operator equation $A\psi = a\psi$ are given by

(i) eigenfunction ψ

(ii) eigenvalue ψ

(iii) eigenvalue a as well as eigen-function ψ

(iv) None of the above

(d) In Schrödinger picture, time evolution operator is

(i) time dependent

(ii) time independent

(iii) constant

(iv) None of the above

(e) The selection rule for radiative transition of an electron in hydrogen atom is

(i) $\Delta l = 0,\ \Delta m_l = \pm 1$

(ii) $\Delta l = \pm 1,\ \Delta m_l = \pm 1$ or 0

(iii) $\Delta l = \pm 1,\ \Delta m_l = 0$

(iv) $\Delta l = \pm 1,\ \Delta m_l = \pm 1$

2. *(a)* What do you understand by quantum mechanical operators? 2

(b) Normalise $\psi(x) = Ae^{-\alpha x^2}$ over the interval $-\infty \le x \le +\infty$. 2

(c) Define Larmor frequency. 2

Or

What are symmetric and anti-symmetric wave functions?

(d) Show that the expectation value of Hamiltonian operator (H) is the total energy (E) of the system. 2

3. *(a)* What is Stark effect? How many components are observed in Stark effect? 1+2=3

Or

Find the magnetic moment, in Bohr magneton, of an atom in the $^3P_2$ state. 3

(b) The ground state of a harmonic oscillator is given by $\psi(x) = Ae^{-\alpha^2 x^2}$ where $\alpha^2 = \dfrac{k}{2\hbar\omega}$, k is the quasistatic force constant ($U = \frac{1}{2}kx^2$). Calculate the mean values of the kinetic and potential energies of the oscillator. 3

Or

Show that for large quantum numbers, the classical probability and quantum mechanical probability of an linear harmonic oscillator become identical. 3

(c) Derive one-dimensional Schrödinger equation from a wave packet and hence show that the energy operator and momentum operator can be represented as follows : 3

$$E \to i\hbar\frac{\partial}{\partial t} \text{ and } P \to i\hbar\frac{\partial}{\partial x}$$

(d) Prove that the relation

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

where $\vec{J}$ in the probability current density and ρ is the probability density. 3

Or

State and prove Heisenberg's uncertainty principle for wave packets. 3

4. *(a)* How do you represent the states of an atom in spectral notion? 4

Or

Show that if $\psi_1(\vec{r})$ and $\psi_2(\vec{r})$ are the independent solutions of the Schrödinger equation, then $\psi(\vec{r}) = \alpha_1\psi_1(\vec{r}) + \alpha_2\psi_2(\vec{r})$ is also a solution. What does it imply? 3+1=4

(b) Explain *L-S* coupling and draw the vector diagram of *LS* coupling. Under what condition *LS* coupling breaks down? 4

5. *(a)* Describe Stern-Gerlach experiment. Discuss how it explained space quantization and electron spin. 5

(b) Write down the Schrödinger wave equation for the hydrogen atom. Separate the Schrödinger equation for the motion of an electron in hydrogen atom into one radial and two angular parts. 5

Or

Show that the orbital angular momentum of the electron in hydrogen atom is $L = \hbar\sqrt{l(l+1)}$, where l is the orbital quantum number. 5

(c) Show that the energy eigenvalues of a particle in a one-dimensional box of infinite depth are discrete and proportional to n^2. Calculate the value of lowest energy of an electron in a one-dimensional force-free region of length 4 Å. 4+1=5

(d) Find the values of $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p_x \rangle$ and $\langle p_x^2 \rangle$ for the Gaussian wave packet

$$\psi(x) = \left(\frac{1}{\sigma\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{x^2}{2\sigma^2}} e^{ik_0 r}$$

5

Or

The wave function of a particle moving in one-dimension is given by

$$\psi(x) = \begin{cases} \sqrt{\dfrac{15}{\alpha}} A(\alpha^2 - x^2), & \text{for } -\alpha \le x \le \alpha \\ 0, & \text{for } |x| > \alpha \end{cases}$$

(i) Find the value of A that will normalise $\psi(x)$.

(ii) Calculate the expectation value of x and p. 3+2=5

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