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3 SEM TDC MTMH (CBCS) C 7

2025

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C–7

(PDE and Systems of ODE)

Full Marks : 60
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Find the degree of the equation

$$x\frac{\partial^2 z}{\partial x^2}+y\left(\frac{\partial z}{\partial y}\right)^{1/3}+kz=0$$ 1

(b) Write Lagrange's subsidiary equation of

$$xzp+yzq=xy$$ 1

(c) Find the complete solution of $p^2+q^2=m$. 1

(d) Form the differential equation of the set of all right circular cones whose axes coincide with z-axis. 5

Or

Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$.

(e) Show that the equations $xp - yq = x$ and $x^2 p + q = xz$ are compatible. 5

Or

Find the integral surface of $x^2 p + y^2 q + z^2 = 0$ which passes through the hyperbola $xy = x + y,\ z = 1$.

2. (a) Find Jacobi's auxiliary equation for 2

$$p_1 x_1 + p_2 x_2 - p_3^2 = 0$$

(b) Find complete integral of any *one* of the following : 4

(i) $q = (z + px)^2$

(ii) $pxy + pq + qy = yz$

(iii) $z^2 = pqxy$

(c) Find complete integral of 6

$$p_3 x_3 (p_1 + p_2) + x_1 + x_2 = 0$$

Or

Solve the boundary value problem $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}$ with $u(x, 0) = x^2 (25 - x^2)$ by the method of separation of variables.

3. (a) Write Laplace equation. 1

(b) Classify the operator 2

$$t \frac{\partial^2 u}{\partial t^2} + 2 \frac{\partial^2 u}{\partial x \partial t} + x \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}$$

(c) Show that $u = f(x + y) + g(y - x)$ satisfies the equation $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} = 0$, where f and g are functions. 2

(d) Derive one-dimensional wave equation. 7

Or

Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} + x^2 \dfrac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

4. (a) Write one assumption of vibrating string problem. 1

(b) Fill in the blank : 1

The partial differential equation in case of vibrating string problem is formulated from the law of ______.

(c) Solve the one-dimensional wave equation by the method of separation of variables. 6

Or

Solve

$$\frac{\partial^2 u}{\partial x^2} = k^2 \left(\frac{\partial u}{\partial t} \right)$$

when $u(0, t) = u(l, t) = 0,\ u(x, 0) = \sin \dfrac{\pi x}{l}$.

5. (a) Write the equation $3 \dfrac{d^2 x}{dt^2} + 6 \dfrac{dx}{dt} - x = t^2$ in normal form. 1

(b) Transform the linear differential equation

$$\frac{d^3x}{dt^3}+2\frac{d^2x}{dt^2}-\frac{dx}{dt}-2x=e^{3t}$$

into system of first-order differential equation. 2

(c) Let $L \equiv D^2+2$, $f(t)=e^{2t}$, where $D \equiv \frac{d}{dt}$. Find $Lf(t)$. 2

(d) Describe the method of successive approximation. 4

Or

Compute $y(0\cdot 2)$ for the differential equation $\frac{dy}{dx}=y^2-x^2$ with $y(0)=1$ using Euler's method.

(e) Using operator method, find the general solution of

$$\frac{dx}{dt}+\frac{dy}{dt}-2x-4y=e^t$$
$$\frac{dx}{dt}+\frac{dy}{dt}-y=e^{4t}$$ 6

Or

Solve :

$$\frac{dx}{dt}=5x-2y$$
$$\frac{dy}{dt}=4x-y$$

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