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3 SEM TDC GEMT (CBCS) GE 3 (A/B/C)

2025

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE–3

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–3A

(Real Analysis)

1. *(a)* Define countable set. 1

(b) Is the set $S = \{2n : n \in \mathbb{N}\}$ denumerable? Justify. 3

(c) Prove that for a set supremum and infimum (if exists) are unique. 4

Or

Find the supremum or/and infinimum of the set $X = \{x \in I / x^2 \leq 36\}$.

(d) State and prove the Archimedean property for real numbers. 4

(e) Prove that the set of natural numbers N is not bounded above. 4

Or

Prove that the real number field $\mathbb{R}$ is an Archimedean field.

(f) State and prove the Bolzano-Weierstrass theorem for sets. 4

2. (a) State true or false : 1

The sequence $\left\{\frac{1}{n}\right\}$ is convergent.

(b) Every convergent sequence is bounded. Is the converse true? Justify. 3

(c) Show that the sequence $\left\{\frac{n+1}{n}\right\}$ converges to 1. 3

Or

Show that every convergent sequence is bounded.

(d) State and prove the Bolzano-Weierstrass theorem for sequences. 5

Or

Define Cauchy sequence. Prove that every sequence is convergent if and only if it is Cauchy sequence.

(e) Show that the sequence $\left\{\frac{3n+7}{4n+8}\right\}$ is monotonic. Is it monotonically increasing or decreasing? 3

Or

Prove that the sequence $\left\{x_n = \frac{n+1}{n}\right\}$ is a Cauchy sequence.

(f) State and prove Cauchy's general principle of convergence of sequence. 5

3. (a) Define infinite series. 1

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$ 5

Or

Prove that the positive term geometric series $1+r+r^2+r^3+\ldots$ converges for $r<1$ and diverges to ∞ for $r \geq 1$.

(c) Prove that the series Σu_n where $u_n = -n$ diverges to $-\infty$. 2

Or

Show that the series $1^2+2^2+3^2+\ldots$ diverges to ∞.

(d) State Cauchy's general principle of convergence of an infinite series. 2

Or

Test the convergence of the series $1+2+3+\ldots$

(e) Test the convergence of any *two* of the following : 5×2=10

(i) $\Sigma\frac{1}{(\log n)^n}$

(ii) $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\cdots$

(iii) $\Sigma[(n+1)^{\frac{1}{2}}-n^{\frac{1}{2}}]$

(iv) $\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}$

4. *(a)* Define alternating series. Show that the series $\Sigma(-1)^{n-1}\frac{1}{x+(n-1)a}$, $x>0$, $a>0$ is convergent. 1+4=5

Or

Define uniform convergence of sequence. Show that the sequence $\{f_n\}$, where $f_n(x)=\frac{x}{1+nx^2}$ is uniformly convergent on any closed interval *I*. 5

(b) Give an example of conditional convergent series. 1

(c) Show that the series

$$\sum_{n=1}^{\infty}\frac{a_n x^n}{1+x^{2n}}$$

is uniformly convergent for all real x if $\sum_{n=1}^{\infty}a_n$ is absolutely convergent. 5

Or

Let $\{f_n\}$ be a sequence of functions such that $\lim_{n\to\infty} f_n(x)=f(x)$, $x\in[a, b]$ and let $M_n=\sup_{x\in[a, b]}|f_n(x)-f(x)|$. Prove that $f_n\to f$ uniformly on $[a, b]$ if and only if $M_n\to 0$ as $n\to\infty$.

(d) If a power series $\Sigma a_n x^n$ converges for $x=x_0$, then prove that it is absolutely convergent for every $x=x_1$ when $|x_1|<|x_0|$. 5

Paper : GE–3C

(Information Security)

1. Answer the following questions : 2×10=20

(a) What type of security was dominant in the early years of computing?

(b) Differentiate between encryption and hashing.

(c) What does two-factor authentication mean?

(d) What is phishing?

(e) Mention two uses of digital signature.

(f) What are plain text and ciphertext in cryptography?

(g) What is Denial of Service (DoS) attack?

(h) What is firewall used for?

(i) Differentiate between threats and attacks.

(j) Mention one difference between symmetric and asymmetric encryption.

2. Answer any *four* of the following : 3×4=12

(a) Explain the three components of the CIA triad with an example for each.

(b) Why is communication privacy important?

(c) What is orange book in information security?

(d) Explain the difference between virus, worm and trojan horse.

(e) What is a zero-day vulnerability? Give a real-world example.

3. Briefly explain encryption and decryption with a suitable example. 6

4. Explain how digital signatures provide authentication, integrity, and non-repudiation. 6

Or

What is Ransomware? Explain its working mechanism and methods of prevention. 2+4=6

5. Briefly explain the principles of information security. 5

6. What is an intrusion detection system? Explain the working principle of a network based intrusion detection system. 2+3=5

Or

What are the challenges in designing an efficient intrusion detection system? 5

7. Discuss the different types of information system audits. 6

Or

Why are system calls important for intrusion detection? Explain how system call monitoring helps in behaviour-based intrusion detection. 2+4=6

8. Answer any *two* of the following : 10×2=20

(a) What is digital signature? Briefly explain the working principle of digital signature. 2+8=10

(b) Explain five different types of information security.

(c) What are the important components of an information system? Explain.

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