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1 SEM TDC GEMT (CBCS) GE 1 (A/B/C)

2019

(December)

MATHEMATICS

(Generic Elective)

The figures in the margin indicate full marks for the questions

Paper : GE–1(A)

(Differential Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) বিছিন্নতাৰ প্ৰকাৰ লিখা যদি $\lim\limits_{x \to a} f(x)$ ৰ অৱস্থিতি থাকে আৰু $f(a)$ ৰ সমান নহয়। 1

Write the type of discontinuity if $\lim\limits_{x \to a} f(x)$ exists but not equal to $f(a)$.

(b) সীমা উলিওৱা (Find) : 2

$$\lim_{x \to \infty} \frac{3x+5}{6x-8}$$

(c) যদি $f(x) = 2x$, $L = 8$, $a = 4$, $\varepsilon = 0 \cdot 1$, δ ৰ মান নিৰ্ণয় কৰা যাতে $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$ হয়। 2

If $f(x) = 2x$, $L = 8$, $a = 4$, $\varepsilon = 0 \cdot 1$, find δ such that $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

2. (a) f ফলন তলত দিয়া দৰে সংজ্ঞাবদ্ধ হয়

$$f(x)=\begin{cases}5x-4, & \text{যেতিয়া } 0<x\le 1\\ 4x^2-3x, & \text{যেতিয়া } 1<x<2\end{cases}$$

তেন্তে দেখুওৱা যে $x=1$ বিন্দুত f ফলন অনৱচ্ছিন্ন। 3

A function f defined as

$$f(x)=\begin{cases}5x-4, & \text{if } 0<x\le 1\\ 4x^2-3x, & \text{if } 1<x<2\end{cases}$$

then show that f is continuous at $x=1$.

(b) দেখুওৱা যে $f(x)=|x|$ ফলন $x=0$ বিন্দুত অৱকলনীয় নহয়। 3

Show that the function $f(x)=|x|$ has no derivative at $x=0$.

(c) প্ৰমাণ কৰা যে যদি $x=c$ বিন্দুত f ফলন অৱকলনীয়, তেন্তে $x=c$ বিন্দুত f অনৱচ্ছিন্ন। 4

Prove that if a function f is differentiable at $x=c$, then f is continuous at c.

3. (a) যদি $y=\log(ax+b)$, তেন্তে y_n নিৰ্ণয় কৰা। 2

If $y=\log(ax+b)$, then find y_n.

(b) লেবনিযৰ উপপাদ্যটো লিখা আৰু প্ৰমাণ কৰা। 4

State and prove Leibnitz's theorem.

অথবা / *Or*

যদি $y=a\cos(\log x)+b\sin(\log x)$, তেন্তে দেখুওৱা যে

$$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$$

If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$$x^2 y_{n+2} + (2n+1)\, x y_{n+1} + (n^2+1)\, y_n = 0$$

(c) দুটা চলকৰ সমমাত্ৰাৰ ফলনৰ বাবে অয়লাৰৰ উপপাদ্যৰ সূত্ৰ লিখা। 1

State Euler's theorem on homogeneous functions of two variables.

(d) যদি $f(x, y) = e^{x^2 + xy}$, তেন্তে $\frac{\partial f}{\partial x}$ উলিওৱা। 1

If $f(x, y) = e^{x^2 + xy}$, then find $\frac{\partial f}{\partial x}$.

(e) যদি $f(x, y) = x\cos y + ye^x$, তেন্তে দেখুওৱা যে

$$f_{xy}(x, y) = f_{yx}(x, y)$$ 3

If $f(x, y) = x\cos y + ye^x$, then show that

$$f_{xy}(x, y) = f_{yx}(x, y)$$

(f) যদি $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, তেন্তে দেখুওৱা যে

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$ 4

If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

অথবা / Or

যদি $u = \sin^{-1} \frac{(x^2 + y^2)}{(x+y)}$, তেন্তে দেখুওৱা যে

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

If $u = \sin^{-1} \frac{(x^2 + y^2)}{(x+y)}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

4. (a) $xy = 2$ সমীকৰণটো ধ্ৰুৱীয় স্থানাংকত প্ৰকাশ কৰা। 1

Express the equation $xy = 2$ in polar coordinates.

(b) $y = x^2 + 1$ বক্ৰটোৰ (2, 5) বিন্দুত স্পৰ্শকৰ নতি উলিওৱা। 1

Find the slope of the tangent to the curve $y = x^2 + 1$ at the point (2, 5).

(c) $x^2 - xy + y^2 = 7$ বক্ৰৰ (–1, 2) বিন্দুত স্পৰ্শকৰ সমীকৰণ উলিওৱা। 3

Find the equation of the tangent to the curve $x^2 - xy + y^2 = 7$ at (–1, 2).

অথবা / Or

$x^2 + xy - y^2 = 1$ সমীকৰণৰ (2, 3) বিন্দুত টনা অভিলম্বৰ সমীকৰণ উলিওৱা।

Find the equation of the normal to the curve $x^2 + xy - y^2 = 1$ at (2, 3).

(d) $\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k},\ a > 0$ কুণ্ডলীৰ বক্রতা নির্ণয় কৰা। 5

Find the curvature for the helix

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k},\ a > 0$$

5. (a) $x = 2t - 3,\ y = 6t - 7$ প্ৰাচলিক সমীকৰণৰ গ্ৰাফ অংকন কৰা। 4

Graph the parametric equations

$$x = 2t - 3,\ y = 6t - 7$$

অথবা / *Or*

$x^2 + (y-3)^2 = 9$ সমীকৰণটো ধ্ৰুৱীয় আকাৰত প্ৰকাশ কৰি গ্ৰাফ অংকন কৰা।

Express the equation $x^2 + (y-3)^2 = 9$ in polar coordinates and then sketch the graph.

(b) ইনফ্লেক্‌চন বিন্দুৰ সংজ্ঞা লিখা। 1

Define inflection point.

(c) $y = \dfrac{x+3}{x+2}$ বক্রৰ অনন্তস্পর্শী নির্ণয় কৰা। 5

Find the asymptote of the curve $y = \dfrac{x+3}{x+2}$.

অথবা / *Or*

$y = x^3 - 3x + 3$ সমীকৰণৰ গ্ৰাফ অংকন কৰা আৰু স্থানীয় চৰম বিন্দু, আৰু ইনফ্লেক্‌চন বিন্দু চিনাক্ত কৰা।

Draw the graph of the equation $y = x^3 - 3x + 3$ and identify local extreme points, and inflection points.

6. *(a)* মধ্যমান উপপাদ্যটো লিখা। 1

Write the statement of mean value theorem.

(b) মধ্যমান উপপাদ্যত $f(b)-f(a)=(b-a)f'(c)$, $a<c<b$, cৰ মান নিৰ্ণয় কৰা, যদি

$$f(x)=x^2+2x-1,\ a=0,\ b=1$$ 3

In the mean value theorem $f(b)-f(a)=(b-a)f'(c)$, $a<c<b$, find the value of c, if

$$f(x)=x^2+2x-1,\ a=0,\ b=1$$

(c) যদি এটা অন্তৰালত xৰ সকলো মানৰ বাবে $f'(x)=0$ হয়, তেন্তে দেখুওৱা যে $f(x)$ সেই অন্তৰালত ধ্ৰুৱক হ'ব। 4

If $f'(x)=0$ for all values of x in an interval, then show that $f(x)$ is constant in that interval.

7. *(a)* ৰোলৰ উপপাদ্যটো লিখা আৰু প্ৰমাণ কৰা। 5

State and prove Rolle's theorem.

(b) ৰোলৰ উপাপাদ্য, $f(x)=x^2-3x+2$ ফলনৰ [1, 2] অন্তৰালত প্ৰতিপন্ন কৰা। 3

Verify Rolle's theorem for the function $f(x)=x^2-3x+2$ in the interval [1, 2].

(c) e^xক xৰ ঘাত হিচাপে মেকলৰিনৰ শ্ৰেণীত বিস্তাৰ কৰা। 3

Expand e^x in powers of x by Maclaurin's series.

8. (a) টেইলৰৰ শ্ৰেণীৰ n তম পদৰ অৱশিষ্ট পদটো লাগ্ৰাঞ্জৰ আকাৰত লিখা। 1

Write the remainder after n terms of Taylor's series in Lagrange's form.

(b) মান নিৰ্ণয় কৰা (যি কোনো দুটা) : 2×2=4

Evaluate (any *two*):

(i) $\lim_{x\to 0} \dfrac{x-\sin x}{x^3}$

(ii) $\lim_{x\to 0} (1+x)^{\frac{1}{x}}$

(iii) $\lim_{x\to \frac{\pi}{2}} \dfrac{1-\sin x}{1+\cos 2x}$

(c) $f(x)$ ফলনৰ $x=a$ বিন্দুত স্থানীয় চৰম মান থকাৰ পৰ্যাপ্ত চৰ্তটো লিখা। 1

Write the necessary condition for a function $f(x)$ to have local extreme value at $x=a$.

(d) মেকলবিনৰ উপপাদ্য ব্যৱহাৰ কৰি $\sin x$ ক x ৰ সূচকত অসীম শ্ৰেণীত বিস্তৃতি কৰা। 5

Using Maclaurin's theorem, expand $\sin x$ in an infinite series in powers of x.

অথবা / *Or*

$f(x)=x^5-5x^4+5x^2-1$ ফলনৰ চৰম মান নিৰ্ণয় কৰা।

Evaluate extreme value of the function

$$f(x)=x^5-5x^4+5x^2-1$$

Paper : GE–1(B)

(Object-oriented Programming in C++)

Full Marks : 60
Pass Marks : 24

Time : 3 hours

1. Answer any *ten* from the following questions : 1×10=10

(a) What are the two characteristics of an object?

(b) What is data abstraction?

(c) In what ways OOP is different from POP?

(d) Define polymorphism.

(e) What are identifiers in C++?

(f) Why is the term 'variable' borrowed from algebra?

(g) What are header files?

(h) How are numbers represented in C++?

(i) What is the difference between array and structure?

(j) What is a function prototype?

(k) Name the different types of storage class specifiers in C++.

(l) What is the symbol of a scope resolution operator?

2. Answer any *three* from the following questions : 2×3=6

(a) Explain the concept of objects, defining the role played by data members and describe the member functions.

(b) What are the advantages of OOP?

(c) What are multidimensional arrays?

(d) What are friend functions? How are they declared?

(e) What is the use of operator overloading?

3. Answer any *three* from the following questions : 4×3=12

(a) Define late binding and abstract class.

(b) What is the use of scope resolution operator? How can a member function be defined outside a class?

(c) Give the advantages of operator overloading. Can all operators available in C++ be overloaded?

(d) What are the various ways in which a base class can be inherited on the basis of access specifier? Give the syntax of each.

(e) Explain the process of declaration of array with an example.

4. Answer any *three* from the following questions : 6×3=18

(a) Write a C++ program that displays factorial of a given number.

(b) Write a C++ program to demonstrate constructor overloading.

(c) Write a C++ program that calculates the *n*th Fibonacci number.

(d) Write a C++ program to demonstrate the multiple inheritances.

5. Answer any *two* from the following questions : 7×2=14

(a) Discuss the various types of inheritances with examples.

(b) Discuss the similarities between a constructor and destructor.

(c) Write short notes on *(i)* destructor and *(ii)* constructor with default arguments.

(d) Define class, member function, object and array.

Paper : GE–1(C)

(Finite Element Methods)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

UNIT—I

1. *(a)* Define finite elements. 1½

(b) Discuss briefly about the applications of finite element method. 4

Or

Mention the areas where finite element methods are applied. Also give reasons of its applicability.

(c) Write down the important features of finite element methods. 3½

(d) Illustrate pictorically the ideal model of finite elements of two degrees of freedom. Is it possible to increase the degrees of freedom? Explain. 3+2=5

2. *(a)* Discuss the merits of finite element methods over finite difference methods. 3

Or

Discuss about different types of partial differential equations and their uses in practical areas.

(b) Discuss about the element characteristic matrix. 3

Or

Mention the process of variational formulation of boundary value problems.

UNIT—II

3. *(a)* Derive an Euler equation from the calculus of variation. 3

Or

How is the functional for a boundary condition of mixed type formulated? Explain.

(b) What do you mean by the shape function? Describe the uses of shape functions in finite element methods. 2+3=5

Or

Illustrate an example to show the formation of the element stiffness matrix.

(c) State the equation of Galerkin's method. Find the formulation of a variation problem. 4

Or

Find the functional in solving the boundary value problem

$$u'' + u = x,\ 0 < x < 1$$
$$u(0) = 0,\ u(1) = 0$$

using the approximate function, $w(x) = x(1-x)(a_1 + a_2 x)$ by Ritz method.

UNIT—III

4. (a) What are linear elements? 2

(b) Formulate a linear Lagrange polynomial and hence find a shape function. 3

(c) Define a quadratic element. Illustrate the physical problems involving quadratic elements. 1+3=4

(d) Discuss about the process of matrix assemblage with example in forming variational function. 3

Or

Write the variational functional for the following boundary value problem :

$$u'' = \frac{3}{2}u^2,\ u(0) = 4,\ u(1) = 1$$

UNIT—IV

5. *(a)* State the Lax-Milgram theorem for elliptic problems. 2

(b) Construct a model for rectangular elements and hence find the element stiffness equations. 3

Or

Find a polynomial for serendipity element. Also geometrically show its affine family.

(c) Define isoparametric elements with example. Find pictorically isoparametric elements with linear and quadratic parametrization. 3

(d) Discuss about sparse matrix calculations with suitable example. Find an algorithm to solve boundary value problems in rectangular elements. 4

Or

Solve the boundary value problem

$$u'' - xu = 0,\ u(0) + u'(0) = 1,\ u(1) = 1$$

with linear piecewise polynomial for two elements of equal length.

UNIT—V

6. *(a)* Define interpolation functions. 1

(b) Compute the element matrices

$$S_{ij}^{01} = \int_0^a \int_0^b \psi_i \frac{d\psi_j}{dx} dxdy, \; S_{ij}^{02} = \int_0^a \int_0^b \psi_i \frac{d\psi_j}{dy} dxdy$$

where ψ_i are the linear interpolation functions of a rectangular element with sides a and b. 5

(c) Develop the linear (3-node) triangular element to higher order triangular elements systematically with the help of Pascal's triangle. 6

Or

Write a note on modeling considerations. Find an algorithm for mesh generation and mesh refinement to get a transition element.

UNIT—VI

7. *(a)* Answer any *one* of the following : 4

(i) Show that area coordinates are related to shape functions for a 3-node triangular element and that area coordinates lead to a different but equivalent, formulation of the shape function.

(ii) A rectangular finite element with dimensions $a \times b$ is defined in an x, y coordinate system for a function $\phi = A + Bx + Cy + Dxy$, derive a shape function.

(b) Derive the local stiffness matrix for plane elasticity for a three-node triangular finite element. 4

(c) A triangular element has node points located at $(x_1 = 1,\ y_1 = 1)$, $(x_2 = 6,\ y_2 = 1)$, $(x_3 = 3,\ y_3 = 4)$. A function has been computed to have nodal point values of $\phi_1 = 900$, $\phi_2 = 600$ and $\phi_3 = 1200$. Using interpolation function for a 3-node triangular element, compute the value of ϕ at $(x = 3,\ y = 4)$. 4

Or

Given the differential equation

$$u\frac{dc}{dx} - D\frac{d^2c}{dx} - m = 0 \text{ with } c(0) = c(4) = 0$$

Assume a solution

$$C_R = a_0 + a_1x + a_2x^2 + a_3x^3$$

Obtain an approximate solution using the Galerkin method.

★★★