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1 SEM TDC PHYH (CBCS) C 1

2019

(December)

PHYSICS

(Core)

Paper : C–1

(Mathematical Physics—I)

Full Marks : 53
Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×3=3

(*a*) The divergence of curl of a vector is

(*i*) 1

(*ii*) 0

(*iii*) $\frac{1}{2}$

(*iv*) $\frac{\pi}{2}$

(b) The condition for a differential equation of the form $Mdx + Ndy = 0$, to be exact is

(i) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

(iv) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(c) The order of a differential equation is always

(i) positive integer

(ii) negative integer

(iii) rational number

(iv) whole number

2. Check whether the function defined by $f(x) = x^2 - \sin x + 5$ is continuous at $x = \pi$. 2

3. (a) Solve the following differential equations (any *two*) : 3×2=6

(i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

(ii) $\frac{dy}{dx} - \frac{y}{x} = 2x$

(iii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x$

(b) State the existence theorem and uniqueness theorem to check whether a solution of a differential equation for a particular boundary value exists or not. 1+1=2

4. (a) Find the partial differentiations $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for the following function : 2

$$f(x, y) = \log(x^2 + y^2)$$

(b) Solve the following differential equations : 2+2=4

(i) $\frac{\partial^2 z}{\partial x^2} = \cos(2x + 3y)$

(ii) $(2x \log x - xy)\, dy + 2y\, dx = 0$

Or

Describe the method of Lagrange's undetermined multipliers for a constrained system. 4

5. (a) If $\vec{A} \times \vec{B} = 0$, is it necessary that $\vec{A}$ and $\vec{B}$ must be parallel? 1

(b) Show that

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$ 2

(c) For vectors $\vec{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, determine the sine of the angle between $\vec{a}$ and $\vec{b}$. 2

6. (a) Evaluate $\iint_S \vec{r} \cdot \hat{n}\, dS$, where S is a surface enclosing a volume V and $\vec{r}$ denotes position vector of a point. 2

(b) Find $\vec{\nabla}\phi$ at the point $(-1, -2, 1)$, where $\phi = x^2 y + xz$. 2

(c) Find a unit vector normal to the surface $z = x^2 + y^2$ at the point $(1, 2, 5)$. 2

(d) Prove that

$$\nabla^2 \left(\frac{1}{r} \right) = 0$$ 3

7. (a) Express Green's theorem in a plane in vector notation. 2

(b) If

$$\vec{v} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k},$$

evaluate $\int \vec{v} \cdot \vec{dr}$ along a straight line from (0, 0, 0) to (1, 0, 0), then to (1, 1, 0) and then to (1, 1, 1). 3

(c) By Stokes theorem prove that

$$\oint \vec{r} \cdot \vec{dr} = 0$$ 4

8. (a) Find the expression for gradient of a scalar function in orthogonal curvilinear coordinates. 3

(b) Express Laplacian in curvilinear coordinates and convert it to cylindrical coordinates. 2

9. What is probability distribution of a random variable? Find the probability distribution for occurrence of a head in tossing a coin twice. Write down the probability distribution function for binomial distribution. 1+2+1=4

Or

What are discrete and continuous probability distributions? Under what condition binomial probability distribution reduces to Poisson's distribution? Write down the probability distribution function for Poisson's distribution. 2+1+1=4

10. Define Dirac delta function. Express it in terms of rectangular function. 1+1=2

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