

1 SEM TDC PHYH (CBCS) C 1

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(March)

PHYSICS

(Core)

Paper : C-1

(Mathematical Physics—I)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer : 1×3=3

(a) The direction of \vec{a} is always

(i) to the surface $\phi = \text{constant}$

(ii) \parallel to the surface $\phi = \text{constant}$

(iii) or \parallel depending upon the shape of surface

(iv) None of the above

(b) The order and degree of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 0$ are

(i) order 2 and degree 2

(ii) order 2 and degree 1

(iii) order 1 and degree 2

(iv) order 3 and degree 2

(c) \vec{A} behaves like

(i) a scalar quantity

(ii) a vector quantity

(iii) scalar or vector

(iv) None of the above

2. Prove that the function $f(x)$ given by

$$f(x) = |x - 1|, x \in \mathbb{R}$$

is not differentiable at $x = 1$.

2

(3)

3. What are linear and non-linear ordinary differential equations? Give examples. 2

4. (a) Solve the following ordinary differential equations (any one) : 3

(i) $(x^2 - y^2)dy = xy dx$

(ii) $\frac{dy}{dx} = \frac{x(y^2 - 1)}{(x - 1)}$

(b) Solve the following 1st order linear differential equations by using integration factor (any two) : 3×2=6

(i) $\frac{dy}{dx} - xy = x$

(ii) $\frac{dy}{dx} - \frac{y}{x} = 2x$

(iii) $x \frac{dy}{dx} - \frac{y}{x} = 3x$

5. Find the partial differentiation

$$\frac{f}{x}, \frac{f}{y}, \frac{\partial^2 f}{\partial x^2} \text{ and } \frac{\partial^2 f}{\partial y^2}$$

for the following functions (any one) : 2

(i) $f(x, y) = x^4 + 2x^2y^2$

(ii) $f(x, y) = 2x^3 + x^2y^6$

(4)

6. Solve the following partial differential equations by the method of separation of variable (any two) : $2^{1/2} + 2^{1/2} = 5$

(i) $3\frac{u}{x} - 2\frac{u}{y} = 0, \quad u = u(x, y)$

(ii) $\frac{u}{x} - 2\frac{u}{t} = u, \quad u = u(x, y)$

(iii) $\frac{u}{x} - 4\frac{u}{y}, \quad u = u(x, y)$

7. (a) What are scalar and vector fields? Give examples. 2

(b) Show that

$$\vec{r} = \frac{1}{r} \frac{\vec{r}}{r^3}$$

where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$. 2

(c) Calculate

$$\vec{r} \cdot \vec{r}$$

if

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z \quad 2$$

(5)

8. (a) What is the geometrical interpretation of gradient of a scalar function? 2

(b) Using Stokes' theorem, prove that

$$\oint_C \vec{r} \cdot d\vec{r} = 0$$

where

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z \quad 5$$

Or

Starting with Maxwell's equation,

$\oint_C \vec{E} \cdot d\vec{r} = \int_V \rho \, dV$, apply Gauss' theorem to

show that

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

(Symbols have their usual meanings.)

(c) Evaluate

$$\oint_S \vec{F} \cdot \hat{n} dS$$

where

$$\vec{F} = (2x - 3z)\hat{i} + (xz - y)\hat{j} + (y^2 - 2z)\hat{k}$$

S being the surface of the sphere having centre (3, 1, 2) and radius 3. 5

(6)

9. (a) Define orthogonal curvilinear coordinates. 2

(b) What is the infinitesimal volume element in spherical polar coordinate system? Compute the volume of a sphere of radius R using the expression for infinitesimal volume element in spherical coordinates. 2+2=4

10. What is Poisson's distribution? Calculate mean of Poisson's distribution. 1+3=4

11. Compute the following integral : 2

$$\int_0^1 (2x - 1) \int_0^1 (1 - x - x^2) dx \, dx$$
