3 SEM TDC MTMH (CBCS) C 7

2020

(Held in April–May, 2021)

MATHEMATICS

(Core)

Paper : C–7

(PDE and Systems of ODE)

Full Marks : 60
Pass Marks : 24

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. (a) Write the order of the equation

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^3=0$$ 1

(b) Write the general solution of the equation of the form $f(p, q)=0$. 1

(c) Choose the correct option : 1

$p\frac{\partial(u, v)}{\partial(y, z)}+q\frac{\partial(u, v)}{\partial(z, x)}=\frac{\partial(u, v)}{\partial(x, y)}$ is a

(i) linear equation

(ii) non-linear equation

(iii) first-order non-linear equation

(iv) second-order equation

(d) Write the form of the general solution of the linear partial differential equation

$$Pp+Qq=R$$ 2

(e) Find the characteristics of the equation $pq=z$. 3

(f) Solve (any *one*) : 4

(i) $zp=-x$

(ii) $p+q=1$

2. (a) Write Charpit's auxiliary equations for the equation $(p^2+q^2)y=qz$. 3

(b) Solve : 5

$$(1+y)p+(1+x)q=z$$

Or

Find the complete integral of $px+qy=pq$.

(c) Solve any *one* by using Jacobi's method : 5

(i) $p^2x+q^2y=z$

(ii) $z^2=pqxy$

3. (a) Write the condition when the equation

$$Rr+Ss+Tt+f(x, y, z, p, q)=0$$

is parabolic. 1

(b) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form. 5

Or

Solve $p + r + s = 1$.

(c) Derive the one-dimensional wave equation. 6

Or

Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

4. *(a)* Write one-dimensional wave equation. 1

(b) Choose the correct option : 1

The equation $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial t}$ is

(i) elliptic

(ii) parabolic

(iii) hyperbolic

(iv) None of the above

(c) Solve one-dimensional heat equation by the method of separation of variables. 6

Or

Describe the physical problem of vibrating string and assumptions on it.

5. *(a)* Write the equation

$$2\frac{dx}{dt} + 6\frac{dy}{dt} + 7y = t$$

in normal form. 1

(b) Transform the linear differential equation

$$\frac{d^2 x}{dt^2} - 3\frac{dx}{dt} + 2x = t^2$$

into a system of first-order differential equations. 3

(c) Let $L_1 \equiv 2D + 1$, $L_2 \equiv D^2 + 1$, $f(t) = t^3$, where $D \equiv \frac{d}{dt}$. Show that $L_1 L_2 f = L_2 L_1 f$. 4

Or

Find the characteristic roots of the equation associated in the solution of

$$\frac{dx}{dt} = 6x - 3y, \quad \frac{dy}{dt} = 2x + y$$

(d) Solve : 7

$$\frac{dx}{dt} + \frac{dy}{dt} - x = -2t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x - y = t^2$$

Or

Find the general solution of the linear system of equations

$$\frac{dx}{dt} = 5x - 2y, \quad \frac{dy}{dt} = 4x - y$$

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