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3 SEM TDC PHYH (CBCS) C 5

2020

(Held in April–May, 2021)

PHYSICS

(Core)

Paper : C-5

(Mathematical Physics—II)

Full Marks : 53
Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) A function $f(x)$ can be expressed in terms of Fourier series, if it is

(i) single valued and periodic

(ii) single valued and non-periodic

(iii) single valued, periodic and bounded

(iv) periodic and bounded

(b) $\sin nx \sin mx\, dx$, if

(i) $n \quad m$

(ii) $n \quad m \quad 0$

(iii) $n \quad m \quad 0$

(iv) $n \quad m$

(c) If $P_n(x)$ be the Legendre polynomial, then $P_n(1)$ is equal to

(i) 0

(ii) 1

(iii) $\frac{1}{2}n(n \quad 1)$

(iv) $\frac{2n}{(n \quad 1)}$

(d) The particular integral in the differential equation $(D^2 \quad ^2)y \quad e \quad ^x$ is

(i) $e \quad ^x$

(ii) $\frac{e \quad ^x}{\quad}$

(iii) $\frac{e \quad ^x}{2}$

(iv) $\frac{e \quad ^x}{2 \quad ^2}$

(e) Relative error is

(i) $\frac{\bar{x}}{x}$

(ii) $\frac{\bar{x}}{x}$

(iii) $\bar{x}$ x

(iv) 0

2. *(a)* Find the series of sines and cosines of multiples of x which represents $f(x)$ in the interval x , where

$f(x)$ 0 when x 0

$\frac{x}{4}$ when 0 x

and hence deduce that

$\frac{^2}{8}$ 1 $\frac{1}{3^2}$ $\frac{1}{5^2}$ ... 5

(b) Find the value of

$_{n\ 1}\frac{1}{n^2}$

using Fourier series. 4

Or

Expand the Fourier series of the periodic function $f(x)$ with period $2l$ which in the interval (l, l) is given by $f(x)$ $|x|$.

3. *(a)* Prove that

$P_n(x)$ $\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2$ $1)^n$

and then find the value of $P_2(x)$. 5

(b) Prove that

$_1^1 P_m(x)P_n(x)dx$ 0 5

Or

$_0^1 x\, J_n(\ x)J_n(\ x)\, dx$ 0

(c) Prove the following relations : 2½×2=5

(i) $(n$ $1)P_{n\ 1}(x)$ $(2n$ $1)xP_n(x)$ $n\ P_{n\ 1}(x)$

(ii) $n\ P_n(x)$ $x\ P\ (x)$ $P_{n\ 1}(x)$

Where the symbols signify usual meaning.

(d) Solve the Bessel's differential equation

$x^2\frac{d^2y}{dx^2}$ $x\frac{dy}{dx}$ $(x^2$ $n^2)y$ 0 5

4. Prove that

(m, n) $\frac{(m)\ \ (n)}{(m\ \ n)}$ 3

5. Discuss the terms *(a)* systematic error, *(b)* random error and *(c)* least count error. 3

6. *(a)* Solve the following partial differential equations : 4×2=8

(i) $\frac{u}{x} \quad 2\frac{u}{t} \quad u$, under the condition $u(x, 0) \quad 6e^{\ 3x}$

(ii) $\frac{u}{x} \quad 4\frac{u}{t}$, under the condition $u(0, y) \quad 8e^{\ 3y}$

(b) Find the solution of Laplace's equation either in 3D Cartesian form or in 3D cylindrical form. 5

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