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3 SEM TDC STSH (CBCS) C 5

2020

(Held in April–May, 2021)

STATISTICS

(Core)

Paper : C–5

(Sampling Distribution)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Find out the correct answer from the following : 1×5=5

(a) A sequence of random variables $X_1, X_2, \cdots, X_n$ is said to converge in probability to a constant a, if for any 0

(i) $\lim_{n} P(|X_n \quad a| \quad) \quad 1$

(ii) $\lim_{n} P(|X_n \quad a| \quad) \quad 1$

(iii) $\lim_{n} P(|X_n \quad a| \quad) \quad 0$

(iv) None of the above

(b) Order statistics are necessarily

(i) independent

(ii) mutually independent

(iii) dependent

(iv) None of the above

(c) The test statistic used for testing the equality of population variance is

(i) ^{2}-test

(ii) *t*-test

(iii) *F*-test

(iv) All of above above

(d) The mean of a chi-square distribution with 12 d.f. is

(i) 12

(ii) 11

(iii) $\sqrt{12}$

(iv) 24

(e) The variance of Student's t-distribution is

(i) n

(ii) $2n$

(iii) $\frac{v}{v \quad 2}$

(iv) None of the above

where v is degrees of freedom

2. Answer the following in brief : 2×5=10

(a) State Chebyshev's inequality.

(b) Distinguish between a parameter and a statistic.

(c) Prove that the sum of two independent chi-square variates is also a chi-square variate.

(d) Write briefly two applications of t-distribution in statistics.

(e) Define type–I error and type–II error.

3. *(a)* Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from a population having continuous distribution function $F(x)$. Define the r-th order statistic $X_{(r)}$ and obtain its cumulative distribution function and hence the probability density function of a single-order statistic. 1+4+3=8

Or

(b) *(i)* State the weak law of large numbers. 2

(ii) Examine whether the weak law of large numbers holds for the sequence $\{X_k\}$ of independent random variables defined as

$P(X_k \quad 2^k) \quad 2^{(2k \quad 1)}$;

$P(X_k \quad 0) \quad 1 \quad 2^{2k}$ 6

4. *(a)* Define ^{2}-statistic and derive its sampling distribution by the method of moment-generating function. Also obtain the moment-generating function of ^{2}-distribution. 6+3=9

Or

(b) If X_1 and X_2 are independent ^{2}-variates with n_1 and n_2 d.f. respectively, then show that U $\frac{X_1}{X_1 \ X_2}$ and V $X_1 \ X_2$ are independently distributed, and U as a $_1(\frac{n_1}{2}, \frac{n_2}{2})$ variate and V as a ^{2}-variate with $(n_1 \ n_2)$ d.f. 5+4=9

5. Derive Snedecor's F-distribution. If $F(n_1, n_2)$ represents an F-variate with n_1 and n_2 d.f., prove that $F(n_2, n_1)$ is distributed as $\frac{1}{F(n_1, n_2)}$. 2+7=9

6. *(a)* What is meant by standard error of a statistic? Write down the standard error of sample mean and sample variance for large random sample.

Obtain the sampling distribution of mean of a random sample drawn from a normal population with mean and variance ^{2}. (1+2)+6=9

Or

(b) Describe the following two tests : 4+5=9

(i) Large sample test for single proportion

(ii) Large sample test for difference of two means

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