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3 SEM TDC STSH (CBCS) C 7

2020

(Held in April–May, 2021)

STATISTICS

(Core)

Paper : C–7

(Mathematical Analysis)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) If the supremum of a set is a limiting point of the set, then it

(i) belongs to the set

(ii) does not belong to the set

(iii) may or may not belong to the set

(iv) None of the above

(b) Every Cauchy sequence must be

(i) monotonic

(ii) bounded above only

(iii) bounded below only

(iv) bounded

(c) According to D'Alembert's ratio test

$$\underset{n}{\text{Lt}} \frac{U_{n \quad 1}}{U_n} \quad l \quad 1$$

means that the series U_n is

(i) divergent

(ii) oscillatory

(iii) convergent

(iv) convergent and to 1 only

(d) One of the conditions for convergence of alternating series

$$\underset{n \quad 1}{} (\quad 1)^{n \quad 1} U_n$$

by Leibnitz test is that

(i) $\underset{n}{\text{Lt}} \; U_n \quad k, (k \quad 0)$

(ii) $\underset{n}{\text{Lt}} \; U_n \quad 1$

(iii) $\underset{n}{\text{Lt}} \; U_n \quad 0$

(iv) $\underset{n}{\text{Lt}} \; U_n \quad 0$

(e) If $f(x)$ is a differentiable function such that $f(x) \quad f(3)$ for $1 \quad x \quad 3$, then

(i) $f\ (3) \quad f(3)$

(ii) $f\ (3) \quad 3$

(iii) $f\ (3) \quad 0$

(iv) $f\ (3)$ does not exist

(f) The function $f(x) \quad 2x^3 \quad x^2 \quad 4x \quad 2$ satisfies all conditions of Rolle's theorem in the interval $[\ \sqrt{2}, \sqrt{2}]$. Then the value of C is

(i) $\quad 1$

(ii) 1

(iii) $\quad 2/3$

(iv) $3/2$

where $(\sqrt{2} \quad C \quad \sqrt{2})$

(g) If $f(x)$ be a polynomial of n-th degree, then

(i) $\ ^{n} f(x) \quad 0$

(ii) $\ ^{n \quad 1} f(x) \quad$ constant

(iii) $\ ^{n \quad 1} f(x) \quad 0$

(iv) $\ ^{n \quad 1} f(x) \quad$ constant

(h) Newton-Raphson method can be used to find

(i) square root of a number

(ii) inverse square root of a number

(iii) cube root of a number

(iv) All of the above

2. Answer the following questions briefly : 2×8=16

(a) Prove that every finite set is bounded.

(b) State Cauchy's general principle of convergence of a series.

(c) Define absolute convergence and conditional convergence of a series.

(d) Define infinite series and positive term series.

(e) Define derivability of a function at a point C and in an interval $[a, b]$.

(f) State the Lagrange's mean value theorem.

(g) Define interpolation and write down the underlying assumptions.

(h) What are the basic conditions to apply Simpson's one-third rule?

3. Answer any *two* of the following : 7×2=14

(a) Define derived sets of a set, open sets and closed sets. Show that derived set of an infinite bounded set is bounded. 3+4=7

(b) Explain limit points of a sequence. Give the characteristic of the supremum and infimum of a bounded sequence. Show that every bounded sequence has a limit point. 2+2+3=7

(c) Define convergent sequences and monotonic sequences. Show, with the help of Cauchy's general principle of convergence, that the sequence f where

$$f(n) \quad 1 \quad \frac{1}{2} \quad \frac{1}{3} \quad \cdots \quad \frac{1}{n}$$

is not convergent. 3+4=7

4. Answer any *two* of the following : 7×2=14

(a) Give a comparison test for positive term series U_n and V_n. Test the convergence of the series $\{(n^3 \quad 1)^{1/3} \quad n\}$. 3+4=7

(b) Define Cauchy's root test and mention about the decision taken when the test fails. Find whether the series

$$2x \quad \frac{3x^2}{8} \quad \frac{4x^3}{27} \quad \cdots \quad \frac{n \quad 1}{n^3} x^n \quad \cdots$$

is convergent or divergent ($x \quad 0$). 3+4=7

(c) What are Raabe's test and Gauss test? Show that the series

$$\frac{1}{1^P} \quad \frac{1}{2^P} \quad \frac{1}{3^P} \quad \frac{1}{4^P} \quad \cdots$$

converges for $P \quad 0$. 3+4=7

5. Answer any *two* of the following : 7×2=14

(a) State Rolle's theorem and mention its applications. Verify Rolle's theorem for $f(x) \quad x^3 \quad 4x$ in $[\quad 2, 2]$. 4+3=7

(b) State the Taylor's theorem with the remainder in Lagrange's form. Expand e^x in a finite series in powers of x with the remainder in Lagrange's form. 3+4=7

(c) Give Maclaurin's expansion for the function $f(x)$ with remainder term. Hence or otherwise show that

$$\log(1 \quad x) \quad x \quad \frac{x^2}{2} \quad \frac{x^3}{3} \quad \frac{x^4}{4} \quad \cdots (\quad 1 \quad x \quad 1)$$

2+5=7

6. Answer any *two* of the following : 7×2=14

(a) Define the operators E, and , and establish that E. Use the method of finite differences to sum the series $1^3 \quad 2^3 \quad 3^3 \quad \cdots \quad n^3$. 4+3=7

(b) When would you recommend the formula involving divided differences and what are the basic differences between divided differences and ordinary differences? Prove that the n-th divided difference can be expressed as the quotient of two determinants, each of order $n \quad 1$. 3+4=7

(c) Define transcendental equation. Give example. How do you proceed to solve such equations? Find the roots of $x^4 \quad x \quad 10 \quad 0$ which is nearer to $x \quad 2$, correct to three places of decimals by using Newton-Raphson method. 1+2+4=7

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