4 SEM TDC MTMH (CBCS) C 8

2022

(June/July)

MATHEMATICS

(Core)

Paper : C–8

(Numerical Methods)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

Use of scientific calculator is allowed

1. *(a)* Write True or False : 1

An exact number may be regarded as an approximate number with error zero.

(b) Define round-off error and truncation error. 1+1=2

(c) The number $x = 37 \cdot 46235$ is rounded off to four significant figures. Compute the absolute error and relative error. 1+1=2

2. (a) Write True or False : 1

A transcendental equation may have no roots.

(b) Find a real root of the equation $x^3 - 3x + 1 = 0$ by the method of bisection correct up to three decimal places. 4

Or

Find a real root of the equation $x^3 - x - 10 = 0$ by the method of secant, correct up to three decimal places.

(c) Describe Newton's method for solution of an algebraic equation. 5

Or

Determine the real root of $2x - 3\sin x - 5 = 0$ by Newton's method correct up to three decimal places.

3. (a) Solve

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

by Gaussian elimination method. 5

Or

Find the solution of the system

$$83x + 11y - 4z = 95$$
$$7x + 52y + 13z = 104$$
$$3x + 8y + 29z = 71$$

by Gauss-Seidel method (obtain three iterations).

(b) Describe Gauss-Jordan method. 5

Or

Solve by Gauss-Jacobi method

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

4. (a) Show that $\Delta - \nabla = \Delta\nabla$. 1

(b) Deduce Lagrange's interpolation formula. 5

(c) Applying Newton's interpolation formula, compute $\sqrt{5 \cdot 5}$ (given that $\sqrt{5} = 2 \cdot 236$, $\sqrt{6} = 2 \cdot 449$, $\sqrt{7} = 2 \cdot 646$, $\sqrt{8} = 2 \cdot 828$). 4

Or

Define interpolation. Write the underlying assumptions for the validity of the various methods used for interpolation.

5. (a) Deduce trapezoidal rule for numerical integration. 5

(b) Evaluate $\int_0^{10} x^2 dx$, by using Simpson's $\frac{1}{3}$ rule. 5

(c) Evaluate the integral of $f(x) = 1 + e^{-x} \sin 4x$ over the interval [0, 1] using Boole's rule (using exactly five functional evaluations). 5

Or

Use the midpoint rule with $M = 5$ to approximate the integral $\int_{-1}^{1} (1 + x^2)^{-1} dx$.

6. (a) Describe Euler's method for first-order and first-degree differential equation. 5

(b) Using the Runge-Kutta method of fourth order, find the numerical solution at $x = 0 \cdot 8$ for $\frac{dy}{dx} = x + y$, $y(0 \cdot 4) = 0 \cdot 41$, assume the step length $h = 0 \cdot 2$. 5

Or

Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 1$, compute $y(0 \cdot 3)$ by Euler's method taking $h = 0 \cdot 1$.

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