4 SEM TDC PHYH (CBCS) C 8

2022

(June/July)

PHYSICS

(Core)

Paper : C-8

(Mathematical Physics—III)

Full Marks : 53
Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×4=4

(a) If z_1 and z_2 are two complex numbers, then

(i) $|z_1 + z_2| \geq |z_1| - |z_2|$

(ii) $|z_1 + z_2| \leq |z_1| - |z_2|$

(iii) $|z_1 + z_2| \leq |z_1| - |z_2| + |z_1 z_2|$

(iv) $|z_1 + z_2| \leq |z_1| + |z_2| + |z_1 z_2|$

(b) The function $f(z) = \frac{1}{(z-2)^3}$ has a/ an _____ at $z = 2$.

(i) essential singularity

(ii) pole

(iii) branch point

(iv) None of the above

(c) The Laplace transform $f(s)$ of $F(t) = t$ is

(i) 1 (ii) s

(iii) s^2 (iv) $1/s^2$

(d) If $g(\omega)$ is the Fourier transform of $f(t)$, then the Fourier transform of $f(at)$ is

(i) $\frac{1}{a} g\left(\frac{\omega}{a}\right)$

(ii) $\frac{1}{\omega} g\left(\frac{\omega}{a}\right)$

(iii) $\frac{1}{\omega} g\left(\frac{a}{\omega}\right)$

(iv) None of the above

2\. (a) Find the polar form of $-5+5i$. 2

(b) Find the residue of the function

$$f(z) = \frac{z}{(z-1)(z+1)^2}$$ 2

(c) Show how Cauchy's theorem can be used for a multiply connected region. 2

(d) Show that the Fourier transform of the derivative of $f(t)$ is $i\omega g(\omega)$, where $g(\omega)$ is the Fourier transform of $f(t)$. 2

(e) Prove that if $f(s)$ is the Laplace transform of $F(t)$, then the Laplace transform of $F(at)$ is

$$\frac{1}{a} f\left(\frac{s}{a}\right)$$ 2

3\. (a) What are the different types of singularities of a complex function? Locate and name the singularities of

$$f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3 (3z+2)^2}$$ 2+3=5

(b) Prove Cauchy-Riemann equations in polar coordinates. 4

Or

If $f(z)$ is an analytic function of z, then prove that

$$\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$$ 4

(c) State the Cauchy's integral formula. Evaluate

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where C is the circle $|z| = 1$. 1+4=5

(d) Find the value of

$$\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}\,dz$$

where C is the circle $|z|=1$. 4

(e) Express the following function in a Laurent's series : 3

$$f(z)=\frac{1}{(z+1)(z+3)}$$

4. Find the Fourier transform of the following functions (any *two*) : 3×2=6

(i) $e^{-|t|}$

(ii) $Ne^{-\alpha x^2}$ (N and α are constants)

(iii) e^{-r^2/a^2} (a is a constant and $r=\sqrt{x^2+y^2+z^2}$)

5. Find the Laplace transform of the following functions (any *two*) : 3×2=6

(i) $t^2 e^t \sin 4t$

(ii) $e^{at}\cos\omega t$

(iii) t^n

6. Write short notes on the following (any *two*) : 3×2=6

(a) Cauchy's theorem

(b) Laplace transforms and its applications

(c) Parseval's theorem

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