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6 SEM TDC DSE MTH (CBCS) 6 (H)

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(June/July)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-6

(**Mathematical Methods**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Write the period of $\sin 2x$. 1
- (b) Write two sufficient conditions for a function to be expanded in a Fourier series. 2
- (c) Find the Fourier series for the function $f(x) = \pi + x$ in the interval $(0, 2\pi)$. 7

(2)

Or

Find the Fourier series for the function $f(x) = x$ in the interval $(0, \pi)$.

2. (a) Write the value of $L\{t^2\}$. 1
 (b) Write the value of $L\{e^{4t}\}$. 1
 (c) Find $L\{\sin 2t\}$. 2
 (d) Find any four of the following : $4 \times 4 = 16$
 (i) $L\{\cosh^2 2t\}$
 (ii) $L\{e^{3t} \cos 2t\}$
 (iii) $L\{(t^2 + 1)^2\}$
 (iv) $L\{(1 + te^t)^3\}$
 (v) $L\{\sin^3 2t\}$
 (vi) $L\{t \sin^2 2t\}$

3. (a) Write the value of $L^{-1}\{2\}$. 1
 (b) Write the value of $L^{-1}\left\{\frac{S}{S^2 + 9}\right\}$. 1
 (c) Find any two of the following : $4 \times 2 = 8$
 (i) $L^{-1}\left\{\frac{1}{(S+1)(S+2)}\right\}$
 (ii) $L^{-1}\left\{\frac{S^2}{(S+2)^3}\right\}$

(3)

$$(iii) L^{-1}\left\{\frac{S}{(S+1)^{\frac{3}{2}}}\right\}$$

$$(iv) L^{-1}\left\{\log \frac{S+6}{S+2}\right\}$$

4. (a) Write the Fourier sine integral formula. 1
 (b) Write the Dirichlet's conditions of Fourier transform. 2
 (c) Answer any three of the following : $5 \times 3 = 15$
 (i) State and prove the change of scale property of Fourier transform.
 (ii) Find the Fourier transform of

$$f(x) = e^{-\frac{x^2}{2}}, -\infty < x < \infty$$

- (iii) Find the inverse Fourier transform of $e^{-|s|y}$.
 (iv) Show that

$$F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} (\tilde{f}(s))$$

- (d) Find the Fourier sine and cosine transforms of $f(x) = x$. 7

Or

Find the Fourier transform of

$$f(x) = \begin{cases} \cos x, & \text{for } |x| < a \\ 0, & \text{for } |x| > a > 0 \end{cases}$$

5. (a) Write the value of $L\left\{\frac{\partial y}{\partial t}\right\}$. 1

(b) Find $L\left\{\frac{\partial y}{\partial x}\right\}$. 2

(c) Solve any two of the following, using Laplace transform : $6 \times 2 = 12$

(i) $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 5y = 5, y(0) = y'(0) = 0$

(ii) $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t,$
 $y(0) = y'(0) = 0$

(iii) $\frac{d^2 y}{dt^2} + y = 0, y(0) = 1, y'(0) = 0$

(iv) $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$
 $x(0) = 2, y(0) = 2$
