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3 SEM TDC PHYH (CBCS) C 5

2022

(Nov/Dec)

PHYSICS

(Core)

Paper : C–5

(Mathematical Physics—II)

Full Marks : 53
Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The value of $erf(x)+erf_c(-x)$ is

(i) 1

(ii) 0

(iii) −1

(iv) None of the above

(b) The value of $\Gamma\left(\frac{1}{2}\right)$ is

(i) $\sqrt{\pi}$

(ii) $\frac{-\pi}{2}$

(iii) $-2\sqrt{\pi}$

(iv) 0

(c) The value of Legendre polynomial $P_{2m+1}(0)$ is

(i) 0

(ii) 1

(iii) 2

(iv) −1

(d) e^{2tx-t^2} is the generating function for

(i) Bessel polynomial

(ii) Laguerre polynomial

(iii) Hermite polynomial

(iv) None of the above

(e) The Fourier series representation of an even function

(i) consists of both sine and cosine terms

(ii) consists of sine terms only

(iii) consists of cosine terms only

(iv) None of the above

2. *(a)* Describe the complex form of Fourier series. 2

(b) Expand the function $f(x) = x + x^2$ in a Fourier series in the interval $-\pi \le x \le \pi$. Hence, show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$ 4+2=6

(c) Represent $f(x)$ in a Fourier series, if

$$f(x) = 1, \quad 0 < x < \frac{1}{2}$$
$$= 0, \quad \frac{1}{2} < x < 1$$ 3

3. *(a)* Determine whether $x = 0$ is an ordinary point or singular point of the following differential equation : 1+2=3

$$2x^2y'' + 7x(x+1)y' - 3y = 0$$

(b) Solve the following using Frobenius method (any *one*) : 5

(i) $x^2y'' + (x + x^2)y' + (x - 9)y = 0$

(ii) $4xy'' + 2y' + y = 0$

(c) Show that $P_n'(1) = \frac{1}{2}n(n+1)$. 3

(d) Show that

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$ 4

Or

Show that

$$\left[J_{\frac{1}{2}}(x)\right]^2 + \left[J_{-\frac{1}{2}}(x)\right]^2 = \frac{2}{\pi x}$$

4. Evaluate : 3

$$\int_0^\infty \sqrt[4]{x} e^{-\sqrt{x}} \, dx$$

Or

Prove that

$$\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$$

5. Answer any *two* of the following : 3×2=6

(a) What are truncation error and rounding off error? Illustrate with examples. 1½×2=3

(b) Find the maximum error in magnitude in the approximation $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ over the rectangle $R: |x - 3| < 0 \cdot 01$ and $|y - 2| < 0 \cdot 01$. 3

(c) What is standard deviation of a data? Calculate the standard deviation of the series $a, a+d, a+2d, \cdots, a+nd$. 3

6. (a) Solve any *two* of the following partial differential equations by method of separation of variables : 4×2=8

(i) $16\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2}$ under the condition $u(x, 0) = x^2(5 - x)$

(ii) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$

(iii) $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$ under the condition $u(x, 0) = 3e^{-5x} + 2e^{-3x}$

(b) Find the solution of 2-D Laplace's equation in spherical polar coordinates. 5

Or

Find the solution of 1-D wave equation by D'Alembert's method.

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