1 SEM TDC PHYH (CBCS) C 1

2022

(Nov/Dec)

PHYSICS

(Core)

Paper : C–1

(Mathematical Physics—I)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) If $z = x^2 + y^2$, then

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2$$

is equal to

(i) $2(x-y)^2$

(ii) $4(x-y)^2$

(iii) 0

(iv) None of the above

(b) The order and degree of the differential equation

$$x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$$

are

(i) 3 and 2

(ii) 2 and 3

(iii) 4 and 3

(iv) None of the above

(c) If $\vec{A}$ is a solenoidal vector, then

(i) $\vec{\nabla}\cdot\vec{A} = 1$

(ii) $\vec{\nabla}\times\vec{A} = 0$

(iii) $\vec{\nabla}\cdot\vec{A} = 0$

(iv) None of the above

(d) By Stokes's theorem

$$\iint_S (\nabla\times\vec{A})\cdot\hat{n}dS$$

is equal to

(i) $\int_S \vec{A}\cdot d\vec{S}$

(ii) $\oint_C \vec{A}\cdot d\vec{r}$

(iii) $\oint_C \vec{A}\cdot d\vec{S}$

(iv) None of the above

(e) $\vec{\nabla}r^n$ is equal to

(i) nr^{n-2}

(ii) $(n-2)r^n\hat{r}$

(iii) $nr^{n-2}\hat{r}$

(iv) $(n-2)r^n$

2. Answer the following questions : 2×5=10

(a) Show that $|x|$ is continuous but not differentiable.

(b) Find the value of m, if $\vec{A} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, $\vec{B} = \hat{i} - m\hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ are coplanar.

(c) If u_p represents orthogonal co-ordinates and h_p represents the corresponding scale factors, then show that

$$|\nabla u_p| = h_p^{-1}$$

(d) Show that Green's theorem in a plane can be expressed as follows :

$$\oint_C \vec{A} \cdot d\vec{r} = \iint_R (\vec{\nabla} \times \vec{A}) \cdot \hat{k} dR$$

(e) Evaluate using property of Dirac delta function

$$\int_{-\infty}^{\infty} e^{-5t} \delta(t-2) dt$$

3. Answer any *five* questions from the following : 4×5=20

(a) What do you mean by integrating factor? Solve the differential equation

$$(x^3 - x)\frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$$ 1+3=4

(b) Solve the following differential equation : 4

$$xdx + ydy = \frac{a^2(xdy - ydx)}{x^2 + y^2}$$

(c) Using Lagrange's method of undetermined multipliers, find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. 3+1=4

(d) Find a unit outward normal drawn to the surface of the paraboloid of revolution $z = x^2 + y^2$ at the point (1, 2, 5). 4

(e) Write the probability distribution functions for Binomial and Poisson distribution. Three distinguishable balls are distributed in three cells. Find the conditional probability that all the three occupy the same cell. Given that at least two of them are in the same cell. 1+3=4

(f) Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F} = x^2\hat{i} + xy\hat{j}$ and C is the boundary of the square in the plane $z = 0$ and bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = a$. 4

4. Answer any *three* questions from the following : 6×3=18

(a) If

$$y_1 = e^{-x}\cos x$$

$$y_2 = e^{-x}\sin x$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

then calculate the Wronskian determinant. Verify that y_1 and y_2 satisfy the given differential equation. Also, check whether y_1 and y_2 are linearly independent. 3+2+1=6

(b) What is directional derivative of a scalar? Find the directional derivative of $\frac{1}{|\vec{r}|}$ in the direction of $\vec{r}$. 1+5=6

(c) State the Gauss divergence theorem. Evaluate

$$\iint_S \vec{F}\cdot\hat{n}\,dS$$

where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$. 1+5=6

(d) Derive the expression for gradient of a scalar in curvilinear co-ordinates. Find the expression for gradient in spherical polar co-ordinates. 3+3=6

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