6 SEM TDC DSE MTH (CBCS) 6 (H)

2 0 2 3

(May/June)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–6

(Mathematical Methods)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. *(a)* If T is the period of the function $f(x)$, then write the period of $f(ax)$, where $a \neq 0$.   1

   (b) Write two Dirichlet conditions for Fourier expansion of a function.   2

   (c) Find a Fourier series for the function $f(x) = x^2$ in the interval $(0, 2\pi)$.   7

   (*Turn Over*)

(2)

Or

Find the Fourier series for the function $f(x) = x \sin x$ in $(-\pi, \pi)$.

2. *(a)* Write the value of *(i)* $L\{t\}$ and *(ii)* $L\{\cos 3t\}$.   1+1=2

   (b) Find $L\{\cos^2 t\}$.   2

   (c) If $f(s) = L\{F(t)\}$, then prove that $L\{e^{at} F(t)\} = f(s-a)$.   4

   Or

   Find $L\{te^{at} \sin at\}$.

   (d) Find any *three* of the following :   4×3=12

      (i) $L\{\sinh^2 t\}$

      (ii) $L\{(t+2)^3\}$

      (iii) $L\{e^{3t} \sin 2t\}$

      (iv) $L\{t\cos^2 t\}$

      (v) $L\{t^2 \cos at\}$

3. *(a)* Write the value of *(i)* $L^{-1}\{\frac{S}{S^2 + 4}\}$ and *(ii)* $L^{-1}\{\frac{1}{S^4}\}$.   1+1=2

   (b) Find $L^{-1}\left\{\frac{S+2}{(S+2)^2 + 6^2}\right\}$.   2

(3)

(c) Find any *two* of the following :   3×2=6

   (i) $L^{-1}\left\{\frac{S}{(S+3)^{7/2}}\right\}$

   (ii) $L^{-1}\left\{\frac{e^{-\pi S}}{S^2 + 4}\right\}$

   (iii) $L^{-1}\left\{\frac{e^{-3S}}{(S-4)^2}\right\}$

   (iv) $L^{-1}\left\{\log \frac{S+4}{S+2}\right\}$

4. *(a)* Write the Fourier cosine integral formula.   1

   (b) Define Fourier transform of a function.   1

   (c) State and prove the linear property of Fourier transform.   5

   Or

   Find the Fourier sine transform of

   $$2e^{-5x} + 5e^{-2x}$$

   (d) Find the Fourier cosine transform of e^{-x^2}.   8

   Or

   Find the Fourier transform of

   $$f(x) = xe^{-x}, 0 \leq x < \infty$$

(e) Find the Fourier transform of

$$f(x) = \begin{cases} \dfrac{1}{2x}, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$$

8

Or

Find the Fourier transform of

$$f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

(f) Write the formula for inverse Fourier transform of a function $f(x)$.

2

5. *(a)* Write the value of $L\left\{\dfrac{\partial^2 y}{\partial t^2}\right\}$.

1

(b) Find the value of $L\left\{\dfrac{\partial^2 y}{\partial x^2}\right\}$.

2

(c) Solve any *two* of the following : 6×2=12

(i) $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} = t^2 + 2t,\ y(0) = 4,\ y'(0) = 2$

(ii) $\dfrac{d^2y}{dt^2} + 25y = 10\cos 5t,$

$$y(0) = 2,\ y'(0) = 0$$

(iii) $t\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + 4ty = 0,\ y(0) = 3,\ y'(0) = 0$

(iv) $\dfrac{\partial y}{\partial x} - \dfrac{\partial y}{\partial t} = 1 - e^{-t},\ 0 < x < 1,\ t > 0$

and $\ y(x,\ 0) = x$

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