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4 SEM TDC PHYH (CBCS) C 8

2023

(May/June)

PHYSICS

(Core)

Paper : C-8

(**Mathematical Physics—III**)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer : 1×4=4

(a) The value of $|3z_1 - 4z_2|$, if $z_1 = 2 + i$
and $z_2 = 3 - 2i$, is

(i) 157

(ii) 167

(iii) $\sqrt{157}$

(iv) $\sqrt{167}$

(2)

(b) The value of the integral $\oint_C \frac{1}{z} dz$, if the point $z = 0$ lies within the curve C , is

(i) $\frac{1}{2\pi i}$

(ii) 0

(iii) $2\pi i$

(iv) None of the above

(c) The Laplace transform $f(s)$ of $F(t) = t$ is

(i) 1

(ii) s

(iii) s^2

(iv) $1/s^2$

(d) If $g(\omega)$ is the Fourier transform of $f(t)$, the Fourier transform of $f(at)$ is

(i) $\frac{1}{a} g\left(\frac{\omega}{a}\right)$

(ii) $\frac{1}{\omega} g\left(\frac{\omega}{a}\right)$

(iii) $\frac{1}{\omega} g\left(\frac{a}{\omega}\right)$

(iv) None of the above

(3)

2. Answer the following :

2×5=10

(a) Express the equation $x^2 + y^2 = 36$ in terms of the conjugate co-ordinates $-5 + 5i$.

(b) Express the complex number $-1 + \sqrt{3}i$ in polar form.

(c) Differentiate between the terms 'essential singularity' and 'removable singularity' with examples.

(d) Using the Fourier integral formula, derive the Fourier cosine transformation.

(e) Prove that if $f(s)$ is the Laplace transform of $F(t)$, the Laplace transform of $e^{at}F(t)$ is $f(s-a)$.

3. Answer the following :

(a) State the Cauchy's residue theorem.

Prove that $\int_0^{\infty} \frac{dx}{x^6 + 1} = \frac{\pi}{3}$. 1+4=5

(b) If $f(z) = u + iv$ is analytic and $u = e^{-x}(x \sin y - y \cos y)$, prove that u is harmonic. Also, find the value of v . 2+3=5

(c) State the Cauchy's integral formula for derivative of a function. Evaluate the integral $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 3$. 1+3=4

(d) Find the residues of the following

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)} \quad 4$$

(e) Find the Laurent series of $\frac{e^{2z}}{(z-1)^3}$

about $z=1$. 3

4. Find the Fourier transform of the following functions (any two) : 3×2=6

(i) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

(ii) $f(x) = Ne^{-\alpha x^2}$ (N and α are constants)

(iii) $f(x) = \frac{1}{2\pi} \left(\frac{a^2}{x^2 + a^2} \right)$

5. Find the Laplace transform of the following functions (any two) : 3×2=6

(i) $f(t) = \frac{e^{at} - 1}{a}$

(ii) $f(t) = \cos^2 t$

(iii) $f(t) = Kt$, where K is a constant

6. Write short notes on the following (any two) : 3×2=6

(a) Singularities of a complex function

(b) Fourier transforms and its applications

(c) Convolution theorem
