Total No. of Printed Pages—20

5 SEM TDC DSE MTH (CBCS)
1.1/1.2/1.3 (H)

2022

(Nov/Dec)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : DSE–1.1

(Analytical Geometry)

1. Answer the following questions :

(a) Write the vertex of the conic

$$(x-1)^2 = 2(y+2)$$ 1

(b) Find the equation of the ellipse whose ends of major axis (0, ±6), and passes through the point (–3, 2). 4

(c) Write the processes to sketch the ellipse. 4

(d) Identify and sketch the curve

$$y^2 - 8x - 6y - 23 = 0$$

and also label the focus, vertex and directrix. 6

Or

Describe the graph of the hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0$$

and sketch its graph.

2. Answer the following questions :

(a) Write the condition of tangency of the line $y = mx + c$ to the parabola $y^2 = 4ax$. 1

(b) Write the reflection property of ellipse. 1

(c) Write the equation of the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$. 1

(d) Derive the equation of tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1). 6

(e) Find the equation of the hyperbola whose length of transverse axis 7 units and foci $(\pm 5, 0)$ and also sketch it. 6

Or

Find and sketch the curve of the ellipse whose foci $(1, 2)$ and $(-1, -2)$ and the sum of the distances from each point $P(x, y)$ on the ellipse is 6 units.

3. Answer the following questions :

(a) Write the condition that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represent a pair of straight lines. 1

(b) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents an ellipse. 1

(c) Determine a rotation angle θ that will eliminate the xy-term of the conic

$$x^2 - 4xy + 4y^2 - 5 = 0$$ 2

(d) Show that the graph of the given equation

$$x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$$

is a hyperbola. Find its foci, vertices and asymptotes. 5

(e) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 60°$.

(i) Find the $x'y'$-coordinate of the point whose xy-coordinate is $(-2, 6)$.

(ii) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$-coordinate. 6

Or

Identify and sketch the curve

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

4. Answer the following questions :

(a) Write the equation of a sphere whose centre is at the origin and radius is r. 1

(b) Write True or False : 1

Curve of intersection of two spheres is a sphere.

(c) Write the standard equation of hyperbola of one sheet. 1

(d) Write the equation of the tangent plane to the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

at $P(x_1, y_1, z_1)$. 2

(e) Find the equation of the sphere passes through the points (0, 0, 0), (0, 1, −1), (−1, 2, 0), (1, 2, 3). 5

(f) A sphere of constant radius k passes through the origin and meets axes in A, B and C. Prove that the centroid of the triangle ABC lies on the sphere

$$9(x^2 + y^2 + z^2) = 4k^2$$ 5

Or

Find the equation of the sphere whose centre at (1, 2, 3) and touching a plane at (2, 1, 3).

5. Answer the following questions :

(a) Find the radius and centre of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,\ x - 2y + 2z = 3$$

5

(b) Find the equation of the sphere whose great circle is

$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0,\ x + y + z = 3$ 5

Or

Prove that the two spheres

$$x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$$

and $$x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$$

touch each other.

6. Answer the following questions :

(a) Find the equation of the two tangent planes to the sphere

$$x^2 + y^2 + z^2 - 2y - 6z + 6 = 0$$

which are parallel to the plane

$$2x + 2y - z = 0$$ 5

(b) Classify and sketch the quadric surface (any *one*) : 5

(i) $36x^2 + 9y^2 + 16z^2 = 144$

(ii) $4x^2 - 3y^2 + 12z^2 + 12 = 0$

Paper : DSE–1.2

(Portfolio Optimization)

1. Answer any *five* of the following questions : 1×5=5

(a) Why do individuals invest?

(b) Write the formula for holding period return (HPR).

(c) What is business risk?

(d) What is security market line (SML)?

(e) What is mutual fund?

(f) Define diversification.

2. *(a)* If a person invests ₹ 200 at the beginning of the year and get back ₹ 220 at the end of the year, find the holding period return (HPR) and holding period yield (HPY) of the investment. 2+2=4

(b) Write two measures of mean historical returns. Calculate the arithmetic mean (AM) of annual holding yields of the investment : 1+2=3

Year	*Beginning Value*	*Ending Value*	*HPY*
1	100·0	115·0	0·15
2	115·0	138·0	0·20
3	138·0	110·4	−0·20

(c) Calculate the risk in terms of variance and standard deviation of the investment in the following scenario : 3+2=5

Economic Condition	Probability	Rate of Return
Strong economy	0·15	0·20
Weak economy	0·15	– 0·20
No major change in economy	0·70	0·10

(d) Discuss the following five risks : 5

(i) Business risk

(ii) Financial risk

(iii) Liquidity risk

(iv) Exchange rate risk

(v) Country risk of an investment

(e) Define risk premium and systematic risk. 2+2=4

(f) Write three ways to change the relationship between risk and the required rate of return for an investment. 4

Or

Write a short note on investment objective and investment constraints.

3. (a) Write two assumptions of the Markowitz's portfolio theory. 2

(b) Find the variance and standard deviation of the following investment scenario : 4

Possible Rate of Return (R_i)	Expected Security Return $E(R_j)$	Probabilities (P_j)
0·08	0·103	0·35
0·10	0·103	0·30
0·12	0·103	0·20
0·14	0·103	0·15

(c) Find the covariance of rates of returns of US stocks and US bonds as given below :

2010	US Stock Index (R_i)	US Bond Index (R_j)
January	– 3·60	1·58
February	3·10	0·40
March	6·03	– 0·85
April	1·58	1·05
May	– 7·99	1·71
June	– 5·24	1·87
July	7·01	0·68
August	– 4·51	2·01
September	8·92	0·02
October	3·81	– 0·16
November	0·01	0·70
December	6·68	– 1·80

If standard deviations of both scenarios are $\sigma_i = 5 \cdot 56$ and $\sigma_j = 1 \cdot 22$, then find the correlation. 4+2=6

(d) State and prove two-fund theorem. 5

Or

Write the assumptions of Capital Market theory.

(e) State one-fund theorem. 2

(f) Write short notes on any *two* of the following : 3×2=6

(i) Optimal portfolio

(ii) Risk-free portfolio

(iii) Efficient frontier

4. (a) What are the values of—

(i) standard deviation of expected return of risk-free asset;

(ii) covariance of any two sets of returns of risk-free asset;

(iii) correlation between risky asset and risk-free asset? 1×3=3

Or

Write a short note on Capital Market Line (CML). 3

(b) Determine the expected rate of return with CAPM for the following five stocks :

Stock	*Beta*
A	0·70
B	1·00
C	1·15
D	1·40
E	−0·30

where economy's PER = 0·05 and expected return on the market portfolio $E(R_M) = 0 \cdot 09$. 5

(c) What is beta of a portfolio? Write the formula for beta of a portfolio. Interpret beta of 1·20 and 0·70. 2+1+2=5

(d) What is security market line? How do you identify that an asset is properly valued, overvalued or undervalued on the graph of Security Market Line (SML)? 2+3=5

Or

Identify the following stocks which are properly valued, overvalued and undervalued : 5

Stock	*Expected Return* $E(R_i)$	*Estimated Return*
A	7·80	8·00
B	9·00	6·20
C	9·60	15·15
D	10·60	5·16
E	3·80	6·00

(e) Suppose that during the most recent 10 years period the average annual total rate of return including dividends on an aggregate market portfolio was 14 percent ($\overline{R}_M = 0 \cdot 14$) and the average nominal rate of return on government *T*-bills was 8 per cent ($\overline{RFR} = 0 \cdot 08$). As administrator of a large pension fund that has been divided among three money managers during the past 10 years. Decide by calculating *T* values whether to renew their investment management contracts based on the following results :

Investment Manager	*Average Annual Rate of Return*	*Beta*
W	0·12	0·90
X	0·16	1·05
Y	0·18	1·20

Also plot their portfolios with security market line (SML). 7

Paper : DSE–1.3

(Financial Mathematics)

(For 2020 batch only)

1. *(a)* Let demand function of an item is represented by $12q + 15p = 190$. Write the inverse demand function. 1

   (b) Among demand and supply functions, write which function changes after introduction of excise tax. 1

   (c) Define equilibrium set for a market. 2

   (d) Define a first-order recurrence. 2

   (e) Describe intervals of compounding. 4

   Or

   Let supply and demand functions for an item are $q^S(p) = bp - a$ and $q^D(p) = c - dp$. If an excise tax *T* per unit is imposed ($T \neq 0$), then find the resulting market price p^T.

2. Answer any *two* from the following questions : 4×2=8

   (a) Describe Cobweb model.

(b) Let supply and demand sets for an item are

$$S = \{(q, p) : 2p - 3q = 12\}$$
$$D = \{(q, p) : 2p + q = 20\}$$

and initial price $p_0 = 10$. Find an expression for the price in the year t.

(c) For the functions

$$S = \{(q, p) : q = bp - a\}$$
$$D = \{(q, p) : q = c - dp\}$$

describe stable and unstable market.

3. (a) Define revenue. 1

(b) Write about inflexion point. 2

(c) Let $I(q) = -14 + 6q - 0{\cdot}2q^2$ be the profit function of a firm which can produce 12 units per day. Find maximum profit. 5

Or

The supply and demand functions are defined by $2q - 5p = 14$ and $3q + 2p = 72$. An excise tax T per unit is imposed. Determine when revenue will be maximum.

4. (a) Write when demand is called inelastic. 1

(b) Define elasticity of demand. 2

(c) Define startup point and breakeven point. 2+2=4

(d) Explain competition versus monopoly. 5

Or

Let the demand is represented by $q = ke^{-m}$, where k, m are constants. Explain elasticity.

5. (a) Explain the three cases how prices of two items may be related to each other. 4

(b) Find and classify the critical points of

$$f(x, y) = x^3 - y^3 - 2xy + 1$$ 6

Or

Find the maximum value of the function

$$f(x, y) = 6 + 4x - 3x^2 + 4y + 2xy - 3y^2$$

6. *(a)* Define arbitrage portfolio. 2

(b) Answer any *two* from the following questions : 5×2=10

(i) Let

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } A^n = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$$

Find recurrence equations for a_n, b_n, c_n and d_n.

(ii) Describe technology matrix.

(iii) Describe a two-industry economy.

7. *(a)* Define cash flow. 1

(b) Define hedging. 1

(c) Write about investment. 2

(d) Describe comparison principle. 2

8. *(a)* Write the alternative name of interest. 1

(b) Define effective interest rate. 1

(c) Write True or False : 1

Effective interest rate and nominal rate are same.

(d) Write the relation between future value and present value. 2

(e) Find the internal rate of return of the cash flow sequence (1, −1, 0, 1). 5

Or

Show that in simple interest, account grows linearly with time.

(f) Describe municipal bonds and callable bonds. 4

Paper : DSE–1.3

(Financial Mathematics)

(For 2019 batch only)

UNIT—I

1. Answer the following as directed : 1×4=4

(a) Write the alternative name of interest.

(b) Define effective interest rate.

(c) Effective interest rate and nominal rate are same.

(Write True or False)

(d) Define discount factor.

2. Answer the following questions : 2×4=8

(a) Write about investment.

(b) Describe comparison principle.

(c) Write risk aversion principle.

(d) Define derivative asset.

3. nswer any *four* of the following questions : 6×4=24

(a) Show that in simple interest, account grows linearly with time.

(b) Show that for a cash flow stream $(x_0, x_1, x_2, \cdots, x_n)$ and an interest r per period the present value is

$$x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + \cdots + \frac{x_n}{(1+r)^n}$$

(c) Find the internal rate of return of the cash flow sequence $(1, -1, 0, 1)$.

(d) Describe municipal bonds and callable bonds.

(e) Write the uses and importance of hedging.

4. Describe comparison principle. 4

UNIT—II

5. Answer the following questions : 1×4=4

(a) Define no-arbitrage assumption.

(b) Write the relation between future value and present value.

(c) Define annuity.

(d) Write when Jensen's index is zero.

6. Answer the following questions : 2×4=8

(a) Write the risk aversion principle.

(b) Define derivative asset.

(c) Write two variations to the generic coupon bond.

(d) Write the linearity property of expected value.

7. Answer any *two* of the following questions : 4×2=8

(a) Compute future value of cash flow stream $(-1, 2, 1, 1\cdot5)$, the periods are years and interest rate is 10%.

(b) Describe price yield curves.

(c) Describe Macaulay duration.

(d) Describe immunization.

8. Answer any *four* of the following questions : 5×4=20

(a) Describe three government securities.

(b) Find the corresponding effective rate for 3%, compounded monthly.

(c) Show that $\frac{dp}{d\lambda} = -D_m P$ with usual notations.

(d) Describe the process of computing internal rate of return.

(e) Describe Markowitz model.

(f) State and describe capital asset pricing model.

★ ★ ★