

5 SEM TDC MTMH (CBCS) C 11

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(Nov/Dec)

MATHEMATICS

(Core)

Paper : C-11

(**Multivariate Calculus**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Let partial derivatives of a function of two variables exist. Does it imply that the function is continuous? 1
- (b) Find $\frac{\partial f}{\partial x}$, where $f(x, y) = e^{x^2 + xy}$. 1
- (c) Show that

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

is continuous at every point, except the origin (0,0). 3

(2)

Or

Using definition, show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

is continuous at the origin.

(d) Find

$$\frac{\partial^3 u}{\partial z \partial y \partial x} \text{ and } \frac{\partial^3 u}{\partial x^2 \partial y}$$

$$\text{if } u = \frac{x}{y+2z}.$$

3

2. (a) Write True or False :

1

"If a function $f(x, y)$ is continuous at (x_0, y_0) , then f is differentiable at (x_0, y_0) ."

(b) Use chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$.

2

(c) Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point (π, π, π) for the function

$$\sin(x+y) + \sin(y+z) + \sin(x+z) = 0$$

4

(3)

Or

Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $\rho_0(1, 1, 0)$ in the direction of $\vec{v} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. In what direction does f increase most rapidly at ρ_0 ?

3. (a) Find the plane, tangent to the surface $z = x \cos y - ye^x$ at $(0, 0, 0)$.

2

(b) Find the local extreme values of

$$f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$$

3

(c) Find the points on the hyperbolic cylinder $x^2 - z^2 = 1$ that are closest to the origin.

5

Or

Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

4. (a) Define gradient vector of $f(x, y)$ at a point.

1

(b) Show that

$$\vec{f}(x, y, z) = (y^2 z^3)\hat{i} + (2xyz^3)\hat{j} + (3xy^2 z^2)\hat{k}$$

is a conservative vector field.

2

(4)

(c) Calculate the curl \vec{f} , where

$$\vec{f} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}} \quad 2$$

5. (a) State Fubini's theorem of first form. 1

(b) Evaluate

$$\iint_R f(x, y) dx dy \text{ for } f(x, y) = 1 - 6x^2y^2,$$

$$R: 0 \leq x \leq 1 \text{ and } -2 \leq y \leq 2. \quad 2$$

(c) Prove that

$$\iint_R e^{x^2 + y^2} dy dx = \frac{\pi}{2}(e - 1)$$

where R is the semicircular region bounded by the x -axis and the curve $y = \sqrt{1 - x^2}$. 3

6. (a) Define volume of a region in space. 2

(b) Find $\int_0^2 \int_0^2 \int_0^2 xyz dx dy dz$. 2

(c) Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$. 5

(5)

Or

Evaluate the following integral by changing the order of the integration in an appropriate way :

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4 \cos(x^2)}{2\sqrt{z}} dx dy dz$$

7. (a) Write the formula for triple integral in spherical coordinates. 1

(b) Evaluate : 4

$$\int_0^\pi \int_0^1 \int_0^{\sqrt{3-r^2}} dz r dr d\theta$$

Or

Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

8. (a) Define Jacobian of a function of two variables. 1

(b) Evaluate : 3

$$\iint_{x^2 + y^2 \leq a^2} (x^2 + y^2) dx dy$$

(c) Find the value of

$$\int_C \{(x+y^2)dx + (x^2 - y)dy\}$$

taken in the clockwise sense along the closed curve C formed by $y^3 = x^2$ and the chord joining $(0,0)$ and $(1,1)$.

3

Or

Evaluate $\int_C (xy + y + z) ds$ along the curve

$$\vec{r}(t) = 2t\hat{i} + t\hat{j} + (2-2t)\hat{k}, \quad 0 \leq t \leq 1.$$

9. (a) Define line integrals of a vector field. 1

(b) Find the circulation of the field

$$\vec{F} = (x-y)\hat{i} + x\hat{j} \quad \text{around the circle}$$

$$\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j}, \quad 0 \leq t \leq 2\pi. \quad 3$$

(c) State and prove the fundamental theorem of line integrals. 4

Or

A fluid's velocity field is $\vec{F} = x\hat{i} + z\hat{j} + y\hat{k}$.

Find the flow along the helix

$$\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j} + t\hat{k}, \quad 0 \leq t \leq \frac{\pi}{2}.$$

10. (a) Define Green's theorem in Tangential form. 1

(b) Evaluate

$$\oint_C (y^2 dx + x^2 dy)$$

using Green's theorem, where C is the triangle bounded by $x=0$, $x+y=1$, $y=0$. 3

(c) State and prove Stoke's theorem. 6

Or

Evaluate $\int_C \vec{F} \cdot d\vec{r}$ by using Stoke's

theorem, if $\vec{F} = x^2\hat{i} + 2x\hat{j} + z^2\hat{k}$ and C is the ellipse $4x^2 + y^2 = 4$ in the xy plane, counterclockwise when viewed from above.

(d) Use Divergence theorem to find the outward flux of \vec{F} across the boundary of the region D , where

$$\vec{F} = (y-x)\hat{i} + (z-y)\hat{j} + (y-x)\hat{k}$$

and D is the cube bounded by the planes $x=\pm 1$, $y=\pm 1$ and $z=\pm 1$. 5
