Total No. of Printed Pages—11

3 SEM TDC GEMT (CBCS) GE 3 (A/B/C)

2023

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE–3

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–3A

(Real Analysis)

1. *(a)* Fill in the blank :
A set S is said to be _____ if it is finite or denumerable. 1

(b) Is the set $E = \{2n : n \in \mathbb{N}\}$ denumerable? Justify. 3

(c) Prove that every subset of a countable set is countable. 4

Or

Prove that union of a finite number of countable sets is countable.

(d) If $a \in \mathbb{R}$ and $a \neq 0$, then prove that $a^2 > 0$. 4

(e) Let S be a non-empty subset of $\mathbb{R}$ that is bounded above and let a be any number in $\mathbb{R}$. If $a+S=\{a+s : s \in S\}$, prove that $\sup(a+S)=a+\sup S$. 4

Or

If $S=\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, prove that $\inf S=0$.

(f) State and prove the nested interval property. 4

2. State True or False : 1

The range of a real sequence may be finite or infinite without ever being the null set.

(b) Every convergent sequence is bounded. Is the converse true? Justify. 3

(c) Write the limit point of the sequence $\{S_n\}$, where

$$S_n=(-1)^n\left(1+\frac{1}{n}\right),\ n \in \mathbb{N}$$

Does the range set have limit points? 2+1=3

(d) State and prove Bolzano-Weierstrass theorem for sequences. 4

Or

Prove that every bounded sequence with a unique limit point is convergent.

(e) Prove that every Cauchy sequence is bounded. Is the converse true? 3+1=4

(f) Show that the sequence $\{S_n\}$, where

$$S_n=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$$

cannot converge. 3

Or

Show that the sequence $\{S_n\}$, where

$$S_n=\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}$$

is convergent for all $n \in \mathbb{N}$.

(g) Is the sequence $\{n^2\}$ a Cauchy sequence? Justify. 2

3. (a) State the necessary condition for the convergence of an infinite series. 1

(b) State Cauchy's general principle of convergence for series. 1

(c) Prove that the positive term geometric series

$$1+r+r^2+r^3+\cdots$$

converges for $r<1$ and diverges to $+\infty$ for $r \geq 1$. 5

Or

Discuss the convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{n^p}$$

(d) Investigate the behaviour of the series whose nth term is

$$\sin\left(\frac{1}{n}\right) \qquad 3$$

Or

Write the three conditions of Leibnitz test.

(e) Test the convergence of any *two* of the following : $5\times2=10$

(i) $\sum_{n=1}^{\infty}\{(n^3+1)^{1/3}-n\}$

(ii) $\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}$

(iii) $\frac{2^p}{1^q}+\frac{3^p}{2^q}+\frac{4^p}{3^q}+\cdots$

(iv) $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\cdots$

4. (a) Test for uniform convergence of the sequence $\{f_n\}$, where

$$f_n(x)=\frac{nx}{1+n^2x^2}, \text{ for all real } x \qquad 5$$

Or

Show that the sequence $\{f_n\}$, where

$$f_n(x)=\frac{x}{1+nx^2}$$

is uniformly convergent on any closed interval I.

(b) Let $\{f_n\}$ be a sequence of functions such that

$$\lim_{n\to\infty} f_n(x)=f(x),\ x\in[a,b]$$

and let $M_n=\sup_{x\in[a,b]}|f_n(x)-f(x)|$

Prove that $f_n\to f$ uniformly on $[a, b]$ if and only if $M_n\to0$ as $n\to\infty$. 5

Or

Let $\{f_n\}$ be a sequence of differentiable functions on $[a, b]$ such that it converges at least at one point $x_0\in[a, b]$. If the sequence of differentials $\{f_n'\}$ converges uniformly to G on $[a, b]$, then prove that the sequence $\{f_n\}$ converges uniformly on $[a, b]$ to f and $f'(x)=G(x)$.

(c) Give an example of a power series. 1

(d) If a power series Σa_nx^n converges for $x=x_0$, then prove that it is absolutely convergent for every $x=x_1$ when $|x_1|<|x_0|$. 5

(e) Determine the radius of convergence and the exact interval of convergence of any *one* of the following : 4

(i) $x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\cdots$

(ii) $1+\frac{3}{5}x+\frac{3.5}{5.10}x^2+\cdots$

Paper : GE–3B

(Cryptography and Network Security)

1. *(a)* Write True or False : 1

Cryptography is used only for encoding the message.

(b) Choose the correct option : 1

In public key encryption, the message is encrypted with the receiver's

(i) private key

(ii) key pair

(iii) symmetric key

(iv) public key

(c) Choose the correct option : 1

In _____, same key is used for encryption and decryption.

(i) symmetric

(ii) asymmetric

(iii) public key

(iv) None of the above

(d) Write True or False : 1

A message digest is also called hash.

(e) What is cryptography? 2

(f) Explain how the private key symmetric encryption works. 10

Or

(g) Describe the RSA algorithm.

(h) Illustrate how digital signature works by giving an example. 8

Or

(i) Briefly explain the SHA-1 algorithm.

2. *(a)* Define two IP SEC protocols. 2

(b) Briefly explain VPN. 8

Or

(c) Explain deniel of service attack.

(d) Define the following (any *three*) : 3×3=9

(i) IP spoofing

(ii) TCP session hijacking

(iii) Sequence guessing

(iv) Teardrop attack

(v) TCP sweeps

(e) Briefly explain how ICMP works. 5

3. *(a)* Briefly explain SNMP architecture. 6

(b) What is firewall? Describe how firewall can be used to protect the network. 2+8=10

Or

(c) Briefly explain the working of secure electronic transaction (SET). 10

(d) Briefly explain the following (any *four*) : 4×4=16

(i) Intrusion Detection System (IDS)

(ii) Encapsulating Security Payload (ESP)

(iii) SSL

(iv) DSS

(v) MAC

(vi) Active attack and passive attack

Paper : GE–3C

(Information Security)

1. Answer any *five* of the following questions : 2×5=10

(a) What is data integrity?

(b) Write the differences between worm and virus in terms of information security.

(c) What is a transpositional cipher?

(d) What is an intrusion detection system?

(e) What is a hash function?

(f) Write the principles of security.

(g) What is a trip-wise security mechanism?

2. *(a)* Compare and contrast substitution and transposition techniques. 3

(b) Briefly describe any *three* of the following : 4×3=12

(i) Trojan horse

(ii) Data availability

(iii) MAC

(iv) Buffer overflow

3. *(a)* Differentiate between symmetric and asymmetric encryptions. 5

(b) Explain Diffie-Hellman key exchange with both keys. Give example. 5+5=10

4. *(a)* Briefly explain the functionalities of data encryption standard (DES). 5

(b) Consider the following :

Plaintext : 'KEY'

SECRET KEY : "CRYPTOGRAPHY"

Compute the cipher text from the given plain text and key using hill cipher method. 5

Or

What are the properties that digital signature should have?

5. *(a)* Write the advantages and disadvantages of secret key encryption. 5

(b) In an RSA system, the public key of a given user is $e = 31$, $n = 3599$. What is the private key of this user? 5

6. *(a)* Generate public key and private key in case of RSA algorithm if two prime numbers given are 5 and 7. ($p = 5$ and $q = 7$) 5

(b) Briefly explain the system threats. 5

7. Explain any *two* of the following : 5×2=10

(a) Auditing and logging

(b) Public key signature

(c) Program threats

(d) Data integrity

★ ★ ★