

3 SEM TDC MTMH (CBCS) C 7

2023

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C-7

(PDE and Systems of ODE)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Write the degree of the equation

$$x^2 p^2 + y^2 r^{\frac{1}{3}} = z^2 \quad 1$$

- (b) Define complete integral of a differential equation. 1

- (c) Find the complete solution of $p^2 + q^2 = m$. 1

- (d) Form the differential equation of the set of all right circular cones whose axes coincide with z-axis. 5

Or

Define quasilinear partial differential

equation. Solve $\frac{y^2 z}{x} dx + xz dy = y^2$.

(e) Solve $pz - qz = z^2 + (x+y)^2$. 5

Or

Find the integral surface of

$$x^2 p + y^2 q + z^2 = 0$$

which passes through the hyperbola $xy = x + y, z = 1$.

2. (a) Write Charpit's auxiliary equations for $q = 3p^2$. 2

(b) Find complete integral of any one of the following : 4

(i) $pxy + pq + qy = yz$

(ii) $z^2 = pqxy$

(iii) $px + qy + pq = 0$

(c) Find a complete integral of $p_1 x_1 + p_2 x_2 = p_3^2$ 6

Or

Solve the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \text{ with } u(x, 0) = x^2(25 - x^2) \text{ by}$$

the method of separation of variables.

3. (a) Write the condition when the equation $Rr + Ss + Tt + f(x, y, z, p, q)$ is elliptic. 1

(b) Classify the operator

$$t \frac{\partial^2 u}{\partial t^2} + 2 \frac{\partial^2 u}{\partial x \partial t} + x \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} \quad 2$$

(c) Show that $u = f(x+y) + g(y-x)$ satisfies the equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ where f and g are functions. 2

(d) Reduce the following equation to canonical form : 7

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$

Or

Derive the one-dimensional heat equation.

4. (a) Write the general form of two-dimensional heat equation. 1

(b) Fill in the blank : 1

The partial differential equation in case of vibrating string problem is formulated from the law of _____.

(c) Solve the one-dimensional wave equation by the method of separation of variables. 6

Or

Find the solution of $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = p_0 \cos pt$ where p_0 is constant when $x = l$ and $y = 0$ when $x = 0$.

5. (a) Give an example of a normal form linear system with variable coefficient. 1

(b) Let $L \equiv D^2 + 2$, $f(t) = e^{2t} + t^2$, where $D \equiv \frac{d}{dt}$. Find $Lf(t)$. 2

(c) Transform the linear differential equation $m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0$ into system of first-order differential equations. 2

(d) Describe Picard method of successive approximations. 4

Or

Compute $y(0.2)$ for the differential equation $\frac{dy}{dx} = y^2 - x^2$ with $y(0) = 1$ using Euler's method.

(e) Solve any one of the following systems : 6

$$(i) \frac{dx}{dt} - \frac{dy}{dt} - 2x + 4y = t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 1$$

$$(ii) \frac{dx}{dt} = 5x - 2y$$

$$\frac{dy}{dt} = 4x - y$$
