3 SEM TDC PHYH (CBCS) C 5

2023

(Nov/Dec)

PHYSICS

(Core)

Paper : C–5

(Mathematical Physics—II)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : $1 \times 5 = 5$

(a) The value of $\text{erf}(\infty)$ is

(i) 1

(ii) 0

(iii) -1

(iv) None of the above

(b) The value of $\Gamma\left(-\frac{1}{2}\right)$ is

(i) $\sqrt{\pi}$

(ii) $\frac{-\pi}{2}$

(iii) $-2\sqrt{\pi}$

(iv) 0

(c) The value of $\int_{-1}^{+1}[P_3(x)]^2 dx$ is

(i) $\frac{2}{3}$

(ii) $\frac{2}{7}$

(iii) $\frac{1}{7}$

(iv) None of the above

(d) The value of Hermite polynomial $H_2(x)$ is

(i) $\frac{1}{2}(3x^2-1)$

(ii) $\frac{1}{3}(3x^2-1)$

(iii) $(4x^2-2)$

(iv) $\frac{1}{2}(4x^2-1)$

(e) According to the Parseval's formula for Fourier series, the integral $\int_{-c}^{c}[f(x)]^2 dx$ equals to

(i) $c\left\{a_0^2+\sum_{n=1}^{\infty}(a_n^2+b_n^2)\right\}$

(ii) $c\left\{\frac{1}{2}a_0^2+\sum_{n=1}^{\infty}(a_n^2+b_n^2)\right\}$

(iii) $c\left\{a_0^2+\frac{1}{2}\sum_{n=1}^{\infty}(a_n^2+b_n^2)\right\}$

(iv) None of the above

2. (a) Describe Dirichlet's conditions for a Fourier series. 2

(b) Expand the function $f(x)$ in a Fourier series where $-\pi < x < \pi$ and $f(x)$ is given by

$$f(x) = 0 \quad \text{when} \quad -\pi < x \le 0$$
$$= \frac{\pi x}{4} \quad \text{when} \quad 0 < x \le \pi$$

Hence show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$ 4+2=6

(c) Obtain the complex form of the Fourier series for the function defined as follows : 3

$$f(x) = 0 \text{ when } -\pi < x \le 0$$
$$= 1 \text{ when } 0 < x \le \pi$$

3. (a) Determine the nature of the point $x = 0$ for $x\frac{d^2y}{dx^2} + y\sin x = 0$. 1+2=3

(b) Solve using Frobenius method (any *one*) : 5

(i) $x^2y'' + (x + x^2)y' + (x - 9)y = 0$

(ii) $4xy'' + 2y' + y = 0$

(c) Express the following in terms of Legendre polynomials : 3

$$4x^2 - 3x + 2$$

(d) Prove that

$$\int_{-1}^{+1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2 - 1}$$ 4

Or

Prove that

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x)H_n(x) = 0 \text{ for } m \ne n$$

4. Evaluate :

$$\int_0^{\infty} x^{n-1} e^{-h^2x^2}\,dx = \frac{1}{2h^n}\Gamma\left(\frac{n}{2}\right)$$ 3

Or

Prove that

$$\int_0^{\pi/2} \sin^p\theta \sin^q\theta\,d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$$

5. Answer any *one* of the following : 6

(a) What is relative error? Describe with an example. $R = \frac{4xy^2}{z^3}$ and errors in x, y, z be 0·001, show that the maximum relative error at $x = y = z = 1$ is 0·006. 1+1+4=6

(b) Describe briefly the least square method of curve fitting. Fit a curve to the following data by the least square method : 2+4=6

x	1	2	3	4
y	1·7	1·8	2·3	3·2

6. *(a)* Solve any *two* of the following partial differential equations by method of separation of variables : 4×2=8

(i) $$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

(ii) $$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \text{ if } u(x, 0) = \frac{1}{2}x(1-x)$$

(iii) $$\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x \text{ given that :}$$

$$u = 0 \text{ when } t = 0$$

$$\frac{\partial u}{\partial t} = 0 \text{ when } x = 0$$

(b) Find the solution of 2-D Laplace's equation in cylindrical polar co-ordinates. 5

Or

Find the vibration modes of a stretched string by solving the one-dimensional wave equation.

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